Imperialist competitive algorithm for minimum bit error rate beamforming

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Abstract: In this paper, the recently introduced optimisation strategy, imperialist competitive algorithm (ICA) is used to design an optimal antenna array which minimises the error probability for binary phase shift keying modulation, called minimum bit error rate (MBER) beamforming. ICA is used to deal with the high complexity and high dimensionality of this challenging problem which can not be easily solved by gradient-based methods. The results are compared to that of both a genetic algorithm (GA) and the optimal array obtained by a properly set gradient based algorithm. Comparison shows that in contrast with GA, ICA leads to the array with error probability which is very close to the optimal value. Also, being faster than GA, ICA minimises the cost function with more consistency. The total results show that ICA is a powerful and reliable tool for solving complex optimisation problems such as MBER beamforming.

Keywords: imperialist competitive algorithm; ICA; minimum bit error rate beamforming; bio-inspired computation.

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1 Introduction

Different methods have been proposed for solving an optimisation problem. Some of these methods are the computer simulation of the natural processes. For example, genetic algorithms (GAs) are a particular class of evolutionary algorithms that evolve a population of candidate solutions to a given problem. GAs use operators inspired by natural genetic variation and natural selection. Simulated annealing (SA) is another example. This technique simulates the annealing process in which a substance is heated above its melting temperature and then gradually cooled to produce the crystalline lattice, which minimises its energy probability distribution. On the other hand, ant colony optimisation is inspired by the foraging behaviour of real ants. Also, the inspiration source of particle swarm optimization (PSO) which was formulated by Edward and Kennedy in 1995 was the social behaviour of animals, such as bird flocking or fish schooling (Haupt and Haupt, 2004).

In this paper, recently introduced evolutionary optimisation strategy, the imperialist competitive algorithm (ICA) is used to solve the beamforming problem in smart antennas. Smart antennas have been widely used in the wireless communication systems and are proposed as solutions to enhance the capacity of the system (Foutz et al., 2003). They are also considerable due to their potential for decreasing the interference, improving quality of service (Bellofiore et al., 2002), enhancing power control and extending battery life in portable units of these systems.

Evolutionary algorithms, such as GA, PSO and SA were suggested in the past decades for solving optimisation problems in different fields. Also, these methods have been recently applied to smart antenna related problems (Zainud-Deen et al., 2005; Li et al., 2008). ICA is a new socio-politically motivated global search strategy that has recently been introduced for dealing with different optimisation tasks (Atashpaz-Gargari and Lucas, 2007). This evolutionary optimisation strategy has shown great performance in both convergence rate and better global optima achievement (Atashpaz-Gargari and Lucas, 2007; Rajabioun et al., 2008a; Biabangard-Oskouyi et al., 2008; Sepehri Rad and Lucas, 2008; Atashpaz-Gargari et al., 2008; Rajabioun et al., 2008b). Nevertheless, its effectiveness, limitations and applicability in various domains are currently being extensively investigated. In Atashpaz-Gargari et al. (2008), ICA is used to design an optimal controller which not only decentralises, but also optimally controls an industrial multi input multi output (MIMO) distillation column process. Biabangard-Oskouyi et al. (2008) use ICA for reverse analysis of an artificial neural network in order to characterise the properties of materials from sharp indentation test. In order to find the optimal priorities for each user in recommender systems, Sepehri Rad and Lucas (2008) use ICA in 'prioritised user-profile' approach to recommender systems, trying to implement more personalised recommendation by assigning different priority importance to each feature of the user-profile in different users. Also, Rajabioun et al. (2008b) use the ICA to find the Nash equilibrium point of different games. In this paper, the ICA is applied to solve the beamforming problem in a smart antenna system in order to minimise the error probability.

The classical approach for beamforming minimises the mean square error between the array output and the desired signal (Chen et al., 2003). This method is called minimum mean square error (MMSE) beamforming. However, in communication applications, the best indicator of the system performance is error probability. Using MMSE criterion does not necessarily guarantee the minimisation of the error probability. Therefore, in communication applications, to obtain further improvement, it is indispensable to minimise the error probability instead of minimising the mean square error. The later method is called minimum bit error rate (MBER) beamforming.

In comparison with MMSE method, employing MBER will require solving a more complicated optimisation problem. To cope with such a hard optimisation task, gradient based methods with proper initial values and algorithmic parameters can be used (Chen et al., 2003) although choosing the initial values and setting the algorithmic parameters is challenging in most of the cases. To avoid the shortcoming of gradient based algorithms, a GA was proposed to solve MBER beamforming problem (Wolfgang et al., 2004). Applying GA to the MBER beamforming has not resulted in solution convergence to the global optimum of the cost function (Wolfgang et al., 2004). This can be caused by the specific shape of cost function, high dimensionality of the optimisation problem as well as the quantised nature of variables in the used GA (Wolfgang et al., 2002). In this paper, ICA is applied to the MBER beamforming to investigate its efficiency.

This paper is organised as follows: Part 2 states the optimisation problem of MBER beamforming. Part 3 provides a review of ICA. To assess the performance of ICA in solving MBER beamforming, in Part 4, simulation results are depicted and finally in Part 5, the conclusion of the paper is presented.

2 Problem statement

As a case study, it is supposed that M users are transmitting their information using binary phase shift keying (BPSK) modulation on the same carrier frequency $w = 2\pi f$. The based band equivalent transmitted signal of ith user can be given by:

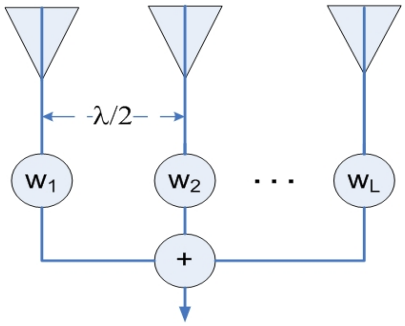
$$m_i(k) = A_i . d_i(k) \qquad i = 1, 2, ..., M, \tag{1}$$

where $d_i(k) \in \{+1, -1\}$ and A_i is the amplitude of the signal received from ith user. It is assumed that the first source is the desired user and the other sources are the interfering users, i.e., to receive the information of the intended user, User 1, other users act as a source of interference.

Figure 1 shows a big picture of an antenna array used in the receiver. As shown in this figure, the receiver is composed of a one-dimensional L-element linear antenna

array with an inter element spacing of $d = \lambda/2$ where λ is the wave-length of the information sources.

Figure 1 Supposed system with L antennas (see online version for colours)



The received signal at lth antenna at the time instance k, $x_l(k)$ can be stated as follows:

$$X_l(k) = \sum_{i=1}^{M} m_i(k) e^{iwt_l(\theta_i)} + n_l(k) \qquad 1 \le l \le L$$
$$= \sum_{i=1}^{M} A_i d_i(k) e^{iwt_l(\theta_i)} + n_l(k), \tag{2}$$

where $t_l(\theta_i)$ is the relative time delay of signal transmitted by the ith user at lth antenna and θ_i is the incidence angle of ith user. Also, $n_l(k)$ is the circularly symmetric additive white Gaussian noise at lth antenna. The mean and variance of the noise $n_l(k)$ is considered to be zero and $2\sigma^2$ respectively. Also, the noises of all antennas are supposed to be independent random variables. The received signals of antennas can be restated in vectorised form as follows:

$$\mathbf{X}(k) = \begin{bmatrix} X_1(k) & X_2(k) & \cdots & X_L(k) \end{bmatrix}^T = P\,\mathbf{D}(k) + \mathbf{n}(k)$$
$$= \mathbf{X}'(k) + \mathbf{n}(k), \tag{3}$$

where P is the system matrix that is defined as follows:

$$P = \begin{bmatrix} A_1\, e^{jw.t_1(\theta_1)} & A_2\, e^{j.w.t_1(\theta_2)} & & A_M\, e^{jw.t_1(\theta_M)} \\ A_1\, e^{jw.t_2(\theta_1)} & A_2\, e^{j.w.t_2(\theta_2)} & & A_M\, e^{jw.t_2(\theta_M)} \\ \vdots & \vdots & \vdots & \vdots \\ A_1\, e^{j.w.t_L(\theta_1)} & A_2\, e^{j.w.t_L(\theta_2)} & & A_M\, e^{jw.t_L(\theta_M)} \end{bmatrix}. \tag{4}$$

$\mathbf{D}(k) = [d_1(k) \quad d_2(k) \quad \ldots \quad d_M(k)]^T$ is bit vector of the users and $\mathbf{n}(k)$ is the additive noise vector for which the covariance matrix is $E(\mathbf{n}(k)\,\mathbf{n}'(k)) = 2\sigma^2 I_L$. $\mathbf{X}'(k)$ is the noiseless input of the antennas for which:

$$\mathbf{X}'(k) \in \tilde{\mathrm{X}} \square \{\bar{\mathbf{X}}_q = P\mathbf{D}_q, 1 \le q \le N_b\},$$

where $N_b = 2^M$ and $\mathbf{D}_q, 1 \le q \le 2^M$ is the set of all the possible sequences for the users bits. In the receiver, the outputs of antennas are multiplied by excitation weights and are summed to yield the output of the antenna array. In the MBER beamformig, the main challenge is to find the proper excitation weights in order to minimise the error probability. The final output of the beamformer is as follows:

$$y(k) = \mathbf{w}^H.\mathbf{X}(k) = \mathbf{w}^H.(\mathbf{X}'(k) + \mathbf{n}(k))$$
$$= \bar{y}(k) + e(k), \tag{5}$$

where $\mathbf{w}$ is the excitation weight vector and $e(k) = \mathbf{w}^H.\mathbf{n}(k)$ is the output noise that has complex Gaussian distribution with zero mean and variance equal to $\mathbf{w}^H.2\sigma^2.\mathbf{w}$. The decision rule for detecting the transmitted bit of the first user is as follows:

$$\hat{b}_1(k) = sign(y_R(k)),$$

where $y_R(k)$ is the real part of the antenna array output and considering:

$$\bar{y}(k) \in Y \square \{\bar{y}_q = \mathbf{w}^H \mathbf{X}_q'(k), 1 \le q \le 2^M\},$$

$\bar{y}_R(k)$ can only take values from the following set:

$$Y_R \square \{\bar{y}_{R_q} = \mathrm{Re}(\bar{y}_q),\ 1 \le q \le 2^M\}.$$

Y_R can be divided into two subsets:

$$Y_R^{(\pm)} \square \{\bar{y}_{R_q}^{(\pm)} \in Y_R,\ d_1(k) = \pm 1\}.$$

It can be easily shown that the probability density functions of $y\ (k)$ can be attained as follows (Chen et al., 2003):

$$p(y_R) = \frac{1}{N_b\sqrt{2\pi\sigma^2 \mathbf{w}^H.\mathbf{w}}} . \sum_{q=1}^{N_b} \exp\left(-\frac{(y_R - \bar{y}_{R_q})^2}{2.\sigma^2 \mathbf{w}^h.\mathbf{w}}\right). \tag{6}$$

Hence, by using probability density function, error probability of the system can be stated as:

$$P_E(\mathbf{w}) = \frac{1}{N_b} . \sum_{i=1}^{N_b} Q\left(\frac{sign(\ d_{q,1}\ )\ \bar{y}_{R_q}}{\sigma\sqrt{\mathbf{w}^h.\mathbf{w}}}\right), \tag{7}$$

where $d_{q,1}$ is the transmitted bit of the first user. The MBER beamforming is the problem of determining optimal excitation weights $\mathbf{w}_{BER}$ which minimise the error probability. That is:

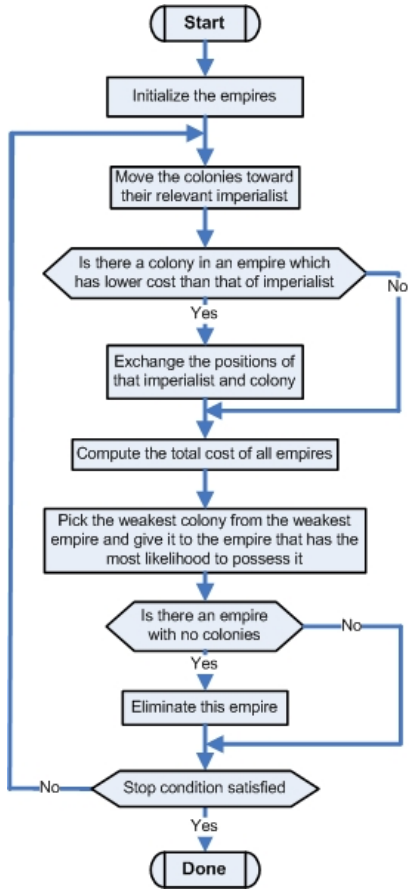
$$\mathbf{w}_{BER} = \arg\min_{\mathbf{w}} P_E(\mathbf{w}).$$

3 Imperialist competitive algorithm

The recently introduced ICA uses the socio-political process of imperialism and imperialistic competition as a source of inspiration. Figure 2 shows the flowchart of the ICA. Similar to other evolutionary algorithms, this algorithm starts with an initial population. Each individual of the population is called a 'country'. Some of the best countries (in optimisation terminology, countries with the least cost) are selected to be the imperialist states and the rest form the colonies of these imperialists. All the colonies of initial countries are divided among the mentioned imperialists

based on their power. The power of each country, the counterpart of fitness value in the GA, is inversely proportional to its cost. The imperialist states together with their colonies form some empires.

Figure 2 Flowchart of the ICA (see online version for colours)



After forming initial empires, the colonies in each of them start moving toward their relevant imperialist country. This movement is a simple model of assimilation policy which was pursued by some of the imperialist states. The total power of an empire depends on both the power of the imperialist country and the power of its colonies. This fact is modelled by defining the total power of an empire as the power of imperialist country plus a percentage of mean power of its colonies.

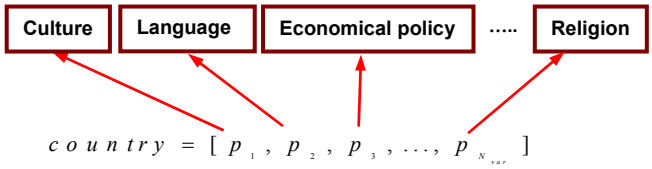
3.1 Creation of initial empires

The goal of optimisation is to find an optimal solution in terms of the variables of the problem. We form an array of variable values to be optimised. In the GA terminology, this array is called 'chromosome', but in ICA, the term 'country' is used for this array. In an N_{var}-dimensional optimisation problem, a country is a $1 \times N_{var}$ array. This array is defined as follow:

$$country = [p_1, p_2, p_3, ..., p_{N_{var}}],$$

where p_is are the variables to be optimised. The variable values in the country are represented as floating point numbers. Each variable in the country can be interpreted as a socio-political characteristic of a country. From this point of view, all the algorithm does is to search for the best country that is the country with the best combination of socio-political characteristics such as 'culture', 'language', 'economical policy' and even 'religion'. From the optimisation point of view, this leads to find the optimal solution of the problem, the solution with least cost value. Figure 3 shows the interpretation of country using some of socio-political characteristics.

Figure 3 The candidate solutions of the problem, called country, consists of a combination of some socio-political characteristics such as culture, language and religion (see online version for colours)



The cost of a country is found by evaluation of the cost function f at variables $(p_1, p_2, p_3, ..., p_{N_{var}})$. So we have:

$$cost = f(country) = f(p_1, p_2, p_3, ..., p_{N_{var}}). \quad (8)$$

To start the optimisation algorithm, initial countries of size $N_{Country}$ is produced. We select N_{imp} of the most powerful countries to form the empires. The remaining N_{col} of the initial countries will be the colonies each of which belongs to an empire.

To form the initial empires, the colonies are divided among imperialists based on their power. That is, the initial number of colonies of an empire should be directly proportionate to its power. To proportionally divide the colonies among imperialists, the normalised cost of an imperialist is defined by:

$$C_n = c_n - \max_i \{c_i\}, \quad (9)$$

where c_n is the cost of the nth imperialist and C_n is its normalised cost. Having the normalised cost of all imperialists, the normalised power of each imperialist is defined by:

$$p_n = \left| \frac{C_n}{\sum_{i=1}^{N_{imp}} C_i} \right|. \quad (10)$$

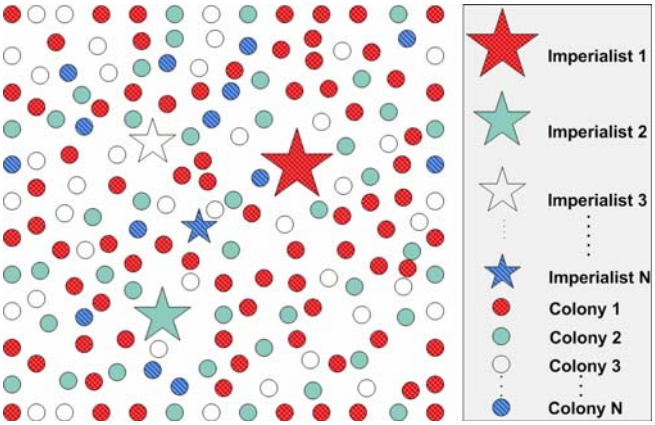
The initial colonies are divided among empires based on their power. Then, the initial number of colonies of the nth empire will be:

$$N.C._n = round\{p_n . N_{col}\}, \quad (11)$$

where $N.C._n$ is the initial number of colonies of the nth empire and N_{col} is the total number of initial colonies. To divide the colonies, $N.C._n$ of the colonies are randomly chosen and given to the nth imperialist. These colonies

along with the *n*th imperialist form the *n*th empire. Figure 4 shows the initial empires. As shown in this figure, bigger empires have greater number of colonies, while weaker ones have less. In this figure, Imperialist 1 has formed the most powerful empire and consequently has the greatest number of colonies.

Figure 4 Generating the initial empires (see online version for colours)



Note: The more colonies an imperialist possess, the bigger is its relevant ★ mark.

3.2 *Assimilation: movement of colonies toward the imperialist*

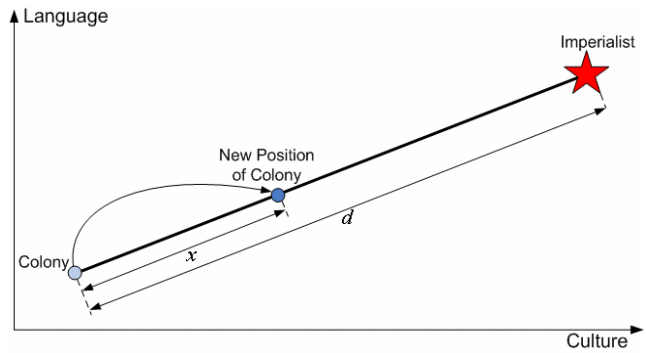
Pursuing assimilation policy, the imperialist states tried to absorb their colonies and make them a part of themselves. More precisely, the imperialist states made their colonies to move toward themselves along different socio-political axis such as culture, language and religion. In the ICA, this process is modelled by moving all of the colonies toward the imperialist along different optimisation axis. Figure 5 shows this movement. Considering a two-dimensional optimisation problem, in this figure, the colony is absorbed by the imperialist in the 'culture' and 'language' axes. Then, the colony will get closer to the imperialist in these axes. Continuation of assimilation will cause all the colonies to be fully assimilated into the imperialist.

In the ICA, the assimilation policy is modelled by moving all the colonies toward the imperialist. This movement is shown in Figure 5 in which a colony moves toward the imperialist by x units. The new position of colony is shown in a darker colour. The direction of the movement is the vector from the colony to the imperialist. In this figure, x is a random variable with uniform (or any proper) distribution. Then:

$$x \sim U\left(0, \beta \times d\right), \tag{12}$$

where β is a number greater than one and d is the distance between the colony and the imperialist state. $\beta > 1$ causes the colonies to get closer to the imperialist state from both sides.

Figure 5 Movement of colonies toward their relevant imperialist (see online version for colours)

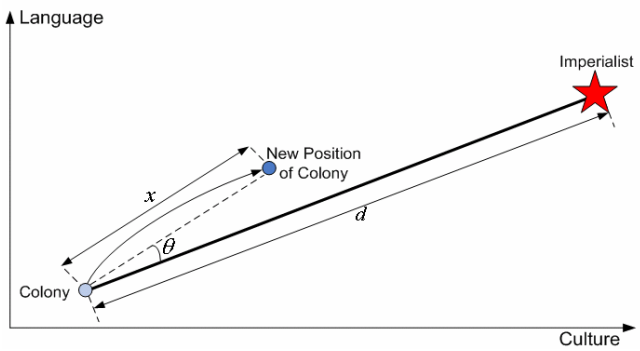


Assimilating the colonies by the imperialist states did not result in direct movement of the colonies toward the imperialist. That is, the direction of movement is not necessarily the vector from colony to the imperialist. To model this fact and to increase the ability of searching more area around the imperialist, a random amount of deviation is added to the direction of movement. Figure 6 shows the new direction. In this figure θ is a parameter with uniform (or any proper) distribution. Then:

$$\theta \sim U\left(-\gamma, \gamma\right), \tag{13}$$

where γ is a parameter that adjusts the deviation from the original direction. Nevertheless, the values of β and γ are arbitrary, in most of implementations, a value of about two for β and about $\pi/4$ (Rad) for γ results in good convergence of countries to the global minimum.

Figure 6 Movement of colonies toward their relevant imperialist in a randomly deviated direction (see online version for colours)

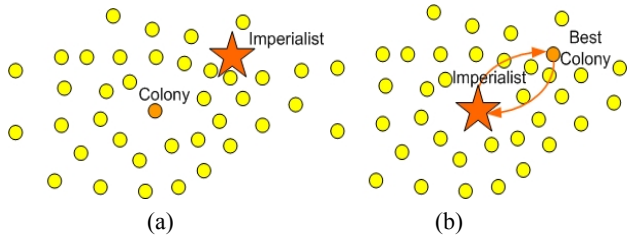


3.3 *Exchanging positions of the imperialist and a colony*

While moving toward the imperialist, a colony might reach to a position with lower cost than the imperialist. In this case, the imperialist and the colony change their positions. Then, the algorithm will continue by the imperialist in the new position and the colonies will be assimilated by the imperialist in its new position. Figure 7(a) depicts the position exchange between a colony and the imperialist. In this figure, the best colony of the empire is shown in a darker colour. This colony has a lower cost than the

imperialist. Figure 7(b) shows the empire after exchanging the position of the imperialist and the colony.

Figure 7 (a) exchanging the positions of a colony and the imperialist (b) the entire empire after position exchange (see online version for colours)



3.4 Total power of an empire

Total power of an empire is mainly affected by the power of imperialist country. However, the power of the colonies of an empire has an effect, albeit negligible, on the total power of that empire. This fact is modelled by defining the total cost of an empire by:

$$T.C. = Cost(imprialist_n) + \xi mean\{Cost(colonies\ of\ empire_n)\}, \tag{14}$$

where $T.C._n$ is the total cost of the nth empire and ξ is a positive small number. A little value for ξ causes the total power of the empire to be determined by just the imperialist and increasing it will increase to the role of the colonies in determining the total power of an empire. The value of 0.1 for ξ has shown good results in most of the implementations.

3.5 Imperialistic competition

All empires try to take the possession of colonies of other empires and control them. The imperialistic competition gradually brings about a decrease in the power of weaker empires and an increase in the power of more powerful ones. The imperialistic competition is modelled by just picking some (usually one) of the weakest colonies of the weakest empire and making a competition among all empires to possess these (this) colonies. Figure 8 shows a big picture of the modelled imperialistic competition. Based on their total power, in this competition, each of the empires will have a likelihood of taking possession of the mentioned colonies. In other words, these colonies will not definitely be possessed by the most powerful empires, but these empires will be more likely to possess them.

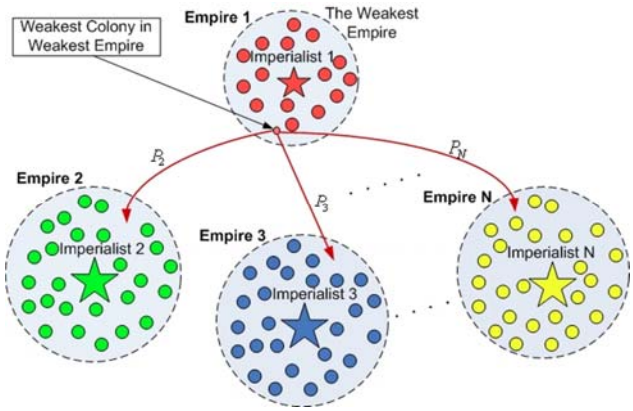
To start the competition, first, a colony of the weakest empire is chosen and then the possession probability of each empire is found. The possession probability P_P is proportionate to the total power of the empire. The normalised total cost of an empire is simply obtained by:

$$N.T.C._n = T.C._n - \max_i\{T.C._i\}, \tag{15}$$

where $T.C._n$ and $N.T.C_n$ are the total cost and the normalised total cost of nth empire, respectively. Having the normalised total cost, the possession probability of each empire is given by:

$$p_{p_n} = \left| \frac{N.T.C._n}{\sum_{i=1}^{N_{imp}} N.T.C._i} \right|. \tag{16}$$

Figure 8 Imperialistic competition (see online version for colours)



Note: The more powerful an empire is, the more likely it will possess the weakest colony of the weakest empire.

To divide the mentioned colonies among empires, vector **P** is formed as follows:

$$\mathbf{P} = \left[p_{p_1}, p_{p_2}, p_{p_3}, \ldots, p_{p_{N_{imp}}} \right]. \tag{17}$$

Then, the vector **R** with the same size as **P** whose elements are uniformly distributed random numbers is created.

$$\mathbf{R} = \left[r_1, r_2, r_3, \ldots, r_{N_{imp}} \right] \quad r_1, r_2, r_3, \ldots, r_{N_{imp}} \sim U(0,1). \tag{18}$$

Then, vector **D** is formed by subtracting **R** from **P**.

$$\mathbf{D} = \mathbf{P} - \mathbf{R} = \left[D_1, D_2, D_3, \ldots, D_{N_{imp}} \right] = \left[p_{p_1} - r_1,\ p_{p_2} - r_2,\ p_{p_3} - r_3\ , \ldots, p_{p_{N_{imp}}} - r_{N_{imp}} \right]. \tag{19}$$

Referring to vector **D**, the mentioned colony (colonies) is handed to an empire whose relevant index in **D** is maximised.

The process of selecting an empire is similar to the roulette wheel process which is used in selecting parents in GA. But this method of selection is much faster than the conventional roulette wheel because it is not required to calculate the cumulative distribution function and the selection is based on only the values of probabilities. Hence, the process of selecting the empires can solely substitute the roulette wheel in GA and increase its execution speed. The main steps of ICA is summarised in the pseudo-code given in Figure 9.

Figure 9 Pseudo code of the ICA

1	Select some random points on the function and initialise the empires.
2	Move the colonies toward their relevant imperialist (assimilation).
3	If there is a colony in an empire which has lower cost than that of the imperialist, exchange the positions of that colony and the imperialist.
4	Compute the total cost of all empires (related to the power of both the imperialist and its colonies).
5	Pick the weakest colony (colonies) from the weakest empires and give it (them) to the empire that has the most likelihood to possess it (imperialistic competition).
6	Eliminate the powerless empires.
7	If there is only one empire left, stop, if not go to 2.

4 Simulation

In this section, two evolutionary optimisation algorithms, ICA and GA are applied to MBER beamforming problem. To simulate a prototype beamforming problem and compare the results with Chen et al. (2003) and Wolfgang et al. (2004), it is assumed that five information sources transmit their signals suing BPSK modulation on the same carrier frequency and the first source is the desired user. The receiver consists of a one-dimensional 4-element antenna array having inter-element spacing of half of the wavelength of the users. Information sources have equal transmit power and the incidence angles of the sources relative to the perpendicular of the antenna array are as follows:

$$\theta_1 = 15^\circ,\ \theta_2 = 4^\circ,\ \theta_3 = 26^\circ,\ \theta_4 = -70^\circ,\ \theta_5 = 80^\circ.$$

As in Chen et al. (2003), the receiver has the knowledge of the location and amplitude of the users. Therefore, the error probability of the detection of the desired (first) user can be written as (7). The main goal in MBER beamforming is to find the optimum excitation weights which minimise the error probability.

Since any positive scaling of excitation weights does not alter the error probability (7), in order to find the optimal weights, the real and imaginary parts of weights are assumed to be in interval [–1, 1].

In this paper, the error probability is minimised using ICA and GA. Also, the optimum excitation weights are found using conjugate gradient method by properly setting the initial values and algorithmic parameters (Chen et al., 2003). The obtained results are also compared with MMSE beamforming method.

Two evolutionary methods are applied to adjust the excitation weights in order to reach the MBER. For a fair comparison between the ICA and GA, initial parameters of the both algorithms are set to have equal numbers of cost function (error probability in terms of excitation weights) calls. In GA, each excitation weight is represented as a string of bits, half of which is used for the real part and the rest for the imaginary part. The parameters of the GA and ICA are summarised in Table 1 and Table 2 respectively.

Table 1 Parameters of GA

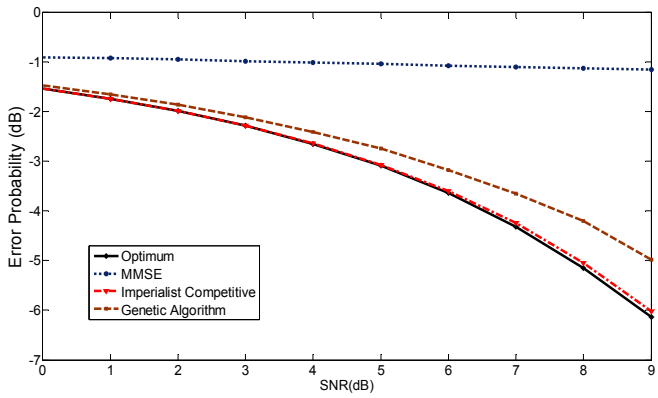
% of cross-over	40
% of recombination	50
% of mutation	10
Population size	40
Number of generations	60

Table 2 Parameters of ICA

Number of countries	40
Number of empires	6
Number of iterations	60
Assimilation coefficient β	2
Assimilation angle γ	0.5

ICA and GA are used to minimise the error probability of the antenna array in different signal to noise ratios (SNR). Each algorithm is run for 50 times and the average error probability is considered. Figure 10 shows the optimum error probability (Chen et al., 2003) and error probability of the ICA and GA versus SNR. As shown in this figure, GA leads to more error probability in comparison with the optimum error probability and the error probability of ICA. This fact becomes more apparent in high values of SNRs. For example, in SNR = 9, error probability of GA is about ten times that of ICA. The performance loss of GA is probably because of the specific shape of the error probability function, high dimensionality of the optimisation problem and the discrete nature of the optimisation variables (Wolfgang et al., 2002). On the contrary, in the same function calls, ICA yields an error probability which is very close to the optimum value.

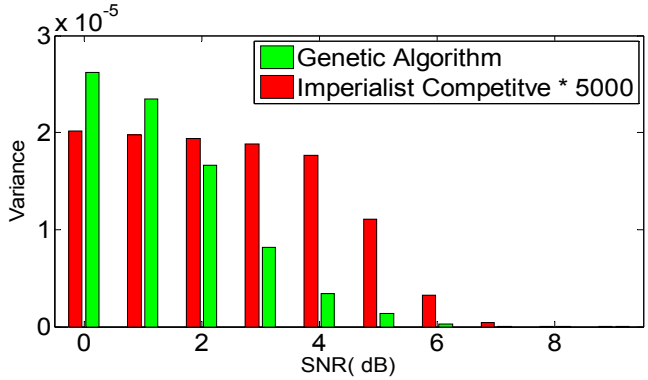
Figure 10 Error probability of antenna array for MMSE, ICA, GA and optimum error probability (see online version for colours)



From another point of view, it is very informative to consider the consistency of the results obtained by ICA and GA. Figure 11 makes a comparison between the variance of the error probabilities of ICA and GA in 50 different runs. To make the figure more apparent, the variance of ICA

results are shown by multiplying them by 5,000. Figure 11 shows that ICA solutions have more consistency in comparison with GA. Figures 10 and 11 indicate that the results of ICA are not only better than that of GA, but also ICA leads to more reliable solutions.

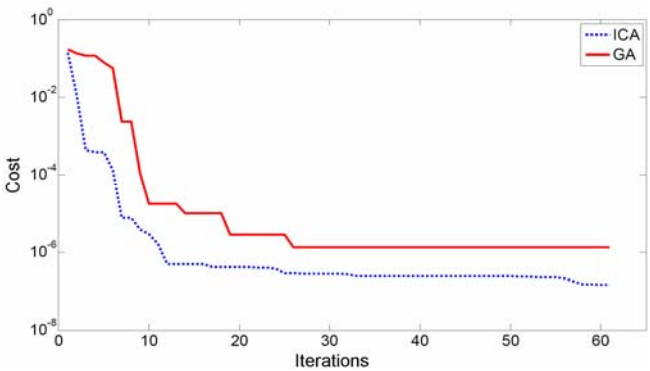
Figure 11 Variances of the results obtained by ICA and GA for different SNRs (see online version for colours)



Note: Variance of ICA is multiplied by 5,000.

Comparing the convergence rate of GA and ICA can be beneficiary too. As a case study, Figure 12 compares the convergence rate of both algorithms for finding the optimal excitation weights for SNR = 10 dB. The comparison results show that ICA not only reaches to less values of error probability, but also its convergence rate is more than that of GA. The convergence rate for other values of SNR showed the similar results.

Figure 12 Minimum costs of ICA countries and GA chromosomes versus iterations (see online version for colours)



5 Conclusions

Beamforming is a promising solution for increasing the capacity of antenna arrays. The classical approach for beamforming was based on the minimisation of the mean square error between the beamformers output and the desired signal. However, in communication applications, a more important criterion is minimising error probability. Therefore, to obtain a better performance in communication applications, minimising the error probability (MBER beamforming) instead of mean square error (MMSE beamforming) is very important and a crucial task.

To deal with the challenging problem of MBER beamforming, in this paper, a recently introduced evolutionary optimisation strategy ICA was applied. The results of applying ICA to MBER beamforming was compared to that of GA and a gradient based method. Comparison of the results showed that ICA has generally reached to an error probability which is very close to the optimum value, while GA's error probability diverges from the optimum value especially in high SNR values. Consistency of the solutions is another criterion in comparing the used optimisation methods. Applying ICA to the problem of MBER beamforming for many times showed that variance of the ICA solutions was very small in comparison with that of GA. The quality of ICA solutions along with their consistency indicates that ICA is an efficient and reliable tool for dealing with difficult optimisation task of MBER beamforming.

References

Atashpaz-Gargari, E. and Lucas, C. (2007) 'Imperialist competitive algorithm: an algorithm for optimisation inspired by imperialistic competition', *IEEE Congress on Evolutionary Computation*, pp.4661–4667.

Atashpaz-Gargari, E., Hashemzadeh, F., Rajabioun, R. and Lucas, C. (2008) 'Colonial competitive algorithm, a novel approach for PID controller design in MIMO distillation column process', *International Journal of Intelligent Computing and Cybernetics*, Vol. 1, No. 3, pp.337–355.

Bellofiore, S., Foutz, J., Govindarajula, R., Bahceci, I., Balanis, C.A., Spanias, A.S., Capone, J.M. and Duman, T.M. (2002) 'Smart antenna system analysis, integration and performance for mobile ad hoc networks (MANET's)', *IEEE Transactions Antennas and Propagation*, Vol. 50, pp.571–581.

Biabangard-Oskouyi, A., Atashpaz-Gargari, E., Soltani, N. and Lucas, C. (2008) 'Application of imperialist competitive algorithm for materials property characterisation from sharp indentation test', to be appeared in the *International Journal of Engineering Simulation*.

Chen, S., Hanzo, L. and Ahmad, N.N. (2003) 'Adaptive minimum bit error rate beamforming assisted receiver for wireless communications', *Acoustics, Speech and Signal Processing*.

Foutz, J., Spanias, A.S., Bellofiore, S. and Balanis, C.A. (2003) 'Adaptive eigen-projection beamforming algorithms for 1-D and 2-D antenna array', *IEEE Antenna and Propagation Letters*, Vol. 2, pp.62–65.

Haupt, R.L. and Haupt, S.E. (2004) *PracticalGeneticAlgorithms*, 2nd ed., John Wiley & Sons, New Jersey.

Li, W.T., Shi, X.W. and Xu, L. (2008) 'Improved GA and PSO culled hybrid algorithm for antenna array pattern synthesis', *Progress in Electromagnetics Research*, PIER 80, pp.461–476.

Rajabioun, R., Atashpaz-Gargari, E. and Lucas, C. (2008a) 'Colonial competitive algorithm as a tool for nash equilibrium point achievement', *Lecture Notes in Computer Science*, Vol. 5073, pp.680–695.

Rajabioun, R., Hashemzadeh, F., Atashpaz-Gargari, E., Mesgari, B. and Rajaei Salmasi, F. (2008b) 'Identification of a MIMO evaporator and its decentralised PID controller tuning using colonial competitive algorithm', accepted to be presented in *IFAC World Congress*.

Sepehri Rad, H. and Lucas, C. (2008) 'Application of imperialistic competition algorithm in recommender systems', in *13th Int'l CSI Computer Conference (CSICC'08)*, Kish Island, Iran.

Wolfgang, A., Ahmad, N.N., Chen, S. and Hanzo, L. (2002) 'Genetic algorithm assisted minimum bit error rate beamforming', *Vehicular Technology Conference*.

Wolfgang, A., Ahmad, N.N., Chen, S. and Hanzo, L. (2004) 'Genetic algorithm assisted error probability optimisation for beamforming', *Electronics Letters*, Vol. 40, No. 5, pp.320–322.

Zainud-Deen, S.H., Mady, E.S., Awadalla, K.H. and Sharshar, H.A. (2005) 'Adaptive arrays of smart antennas – using genetic algorithm', *22nd National Radio Science Conference (NRSC 2005)*, Cairo, Egypt, pp.145–154.