World Applied Sciences Journal 14 (10): 1576-1598, 2011
ISSN 1818-4952

Application of an Imperialist Competitive Algorithm in Portfolio Optimization

Arash Talebi, Mohammad Ali Molaei and Bozorgmehr Ashrafi

Department of Industrial Engineering and Management,
Shahrood University of Technology, Shahrood, Iran

Abstract: The aim of this study is to help finance practitioners/researchers with the portfolio problem, which deals with simultaneous *risk minimization/return maximization* as the objective. This paper proposes a new approach based on the recently introduced Imperialist Competitive Algorithm (ICA) - a novel optimization algorithm inspired by *socio-political* process of imperialistic competition - to resolve the portfolio problem. Security's weights are represented by *"countries"* and manipulated for optimization. Its performance is evaluated using real market data. An ICA-based portfolio is selected from Tehran Stock market; thereafter, the portfolio is applied to the market for a six month period - the *"test period"*. The results indicate the ICA-based portfolio has a better performance in contrast to *the market portfolio*. It outperforms the return of the *market portfolio* often during the test period. The approach is shown to be efficient; it can locate the solution in complicated/large search space of the real market with fast convergence speed, constant iteration number/time and never misses the optimal solution even when run repeatedly. The parameters of the proposed approach are calibrated throughout paper; the parameters that are consistent with the problem are reported. The approach effectiveness is proven; the results indicate it can consistently handle the practical portfolio problem.

Key words: Evolutionary and heuristic optimization · Imperialist Competitive Algorithm (ICA) · Modern portfolio theory (MPT) · Portfolio management · Investment decisions · Stock Markets
JEL *Classification:* C61 · C63 · G11

INTRODUCTION

Contrary to the growing use of portfolios and in spite of the rich literature on the subject, there are some problems and unanswered questions. The rationale for stock market diversification is that the overall risk from owning many stocks is lower than the risk of holding a few stocks [1]. n a metaphor *"Money is like manure - when it stacks up, it stinks; when you spread it around, it makes things grow."* Texan Clint Murchison, Sr., one of the wealthiest investors ever.

Effects of diversification on risk reduction have been studied extensively [2-4], but the importance of portfolio management lies not in the number of holdings, but rather in both the nature and degree of the combined risk of the underlying stocks [1,2] has shown quite dramatically what can happen when one considers both the nature and degree of risk in portfolio composition. He reported that a portfolio containing only 11 securities, which were *carefully selected* for their risk-diversifying characteristics, would be less risky than a portfolio of 2000 securities, which were selected *without regard* to risk! So the question is *how should investment risk be considered?* Furthermore, *once it is considered, how should portfolios be optimized?* This paper intends to summarize the answers to the above questions *efficiently* and also to make new viewpoints on the answers to the second question.

Modern portfolio theory (MPT) takes its origin from the pioneering work of Markowitz. Since Markowitz's 1952 revolutionary article on portfolio selection [5], there has been many contributions to this important field of financial studies and within last 60 years considerable progress has been made. Markowitz's work has permanently changed the course of investment-related thought. Before Markowitz's article, it was more or less taken for granted that the proper way to construct an investment portfolio was just to select the best securities. It was erroneously assumed that this technique would maximize the expected return of the resultant portfolio.

Corresponding Author: Arash Talebi, Apartment number 25, Sarv Building, Building number 48, 11th Alley, South Ibrahimi Pour Basayi Street, Ferdows Boulevard, Sadeghieh's 2nd Square, Tehran, Iran. Postal Code: 1481955437. Tel/Facsimile: +98-21-44098164, E-mail: Arash.talebi.mba@gmail.com.

Without the benefit of Markowitz's insights, dangerous homilies such as *"put all your eggs in one basket and watch the basket!"* still prevail [6].

The original Markowitz model of portfolio selection [5] has received a widespread acceptance and it has been the basis for various portfolio selection techniques. Markowitz model has received remarkable attention of math and computer science experts, because finding the optimum solution to the model equations has always been a challenging issue. Traditionally, the portfolio selection problem has been handled by trial and error or via classic approaches, also referred to as *gradient-based algorithms*, like *"quadratic programming"*. In the past decades, more reliable methods based on evolutionary and heuristic approaches - due to their advantages over their classic counterparts in portfolio selection [7] have been applied to the problem. The era of evolutionary computation started with genetic algorithms (GA), more than three decades ago [8]. Amounts of applications have benefited from the utilization of GA [9]; as one of the earliest evolutionary computation techniques, GA and its different types have been widely employed to solve the portfolio selection problem (For a list of related researches see [10-13] [14, 15] [16, 17]). Moreover, other heuristic approaches have also been used to solve the portfolio selection problem with different criteria (For a list of related researches see [18-34]).

The GA approach searches for the optima from a lot of generations of the chromosome-represented schedules that are reproduced through cross-over and mutation, without referring to any heuristic rules. The internal updating mechanism of chromosomes enables GA to search for the global optimum by escaping from local optima. Therefore, the GA approach can overcome the drawbacks of the analytical and heuristic approaches. Nevertheless, some deficiencies in GA performance, including premature convergence or slow convergence process (*i.e.*, requiring a large number of time-taking generations) [35] and the unpredictability of the result have also been identified [36]. This degradation in efficiency is apparent in applications with highly *epistatic* objective functions (*i.e.* where the parameters being optimized are highly correlated). The genetic operators alone cannot ensure better fitness of offspring because chromosomes in the population have a similar structure and their average fitness are high toward the end of the evolutionary process [37].

Recently, an optimization algorithm has been developed based on the simulation of *socio-political evolution processes, Imperialist Competitive Algorithm (ICA, also known by the name Colonial Competitive Algorithm or CCA)* [38]. This paper describes and applies this novel socio-politically inspired evolutionary optimization algorithm to the portfolio optimization problem. Unlike the current evolutionary algorithms such as genetic algorithm (GA), simulated annealing (SA) [39], Particle Swarm optimization (PSO) [40]; [41] and ant colony [42] that are computer simulation and mimic of natural processes[1], *ICA* uses imperialism and imperialistic competition as the source of inspiration, a *non*-natural source of inspiration. ICA is computationally efficient and has a great capability of escaping local optima.

Similar to the other evolutionary algorithms that start with an initial population, ICA begins with *"initial empires"*. Any individual of an empire is called a *"country"*. There are two types of countries; colonies and imperialist states that collectively constitute empires. Imperialistic competitions among these empires along with assimilation policy constitute the basis of the ICA. During this competition, weak empires collapse and powerful ones take possession of the colonies of collapsed empires. Imperialistic competitions converge to a state in which there exists only one empire and its colonies are in the same position and cost as with the imperialist.

This evolutionary optimization strategy has shown great performance in both convergence rate and global optimum achievement. Nevertheless, because it is a perfectly new method, its effectiveness, limitations and applicability in various domains are being currently extensively investigated. Applications of ICA in electrical and communications engineering problems, which are the first applications of ICA, include - but not limited to -[43], which use ICA to design an optimal controller that not only decentralizes, but also optimally controls an industrial Multi Input Multi Output (MIMO) Evaporator system, [44] applies ICA to the design of a linear induction motor, [45] uses ICA for localization in wireless sensor networks, [46] invokes ICA for Image Clustering purposes. In the fields of geology and civil engineering, ICA is used for prediction of horizontal peak ground and spectral accelerations [47] and also the design of skeletal structures [48]; [49]. Material engineers have exercised ICA for material characterization [50]. In pure optimization category, [51] uses ICA to optimize ANN [2] weights. Also, Fuzzy time series based on defining interval length is

[2]Artificial Neural Network

optimized via ICA [52], another application of ICA in fuzzy systems was for the design of a Fuzzy controller in vehicles [53]. As an economic application, [54] employed ICA as a tool for Nash Equilibrium point achievement, in Game theory. Another economical application of ICA was introduced in the simulation of Energy demand based on economic indicators [55]. Computer science problems, have also invoked ICA; in order to find the optimal priorities for each user in recommender systems, [56] uses ICA in *"Prioritized user-profile"* approach to recommender systems, trying to implement a more personalized recommendation by assigning different priority importance to each feature of the user-profile in different users, [57] uses ICA for Mobile Robot Global Localization. Other applications of this novel algorithm could be found in Industrial engineering and managerial problems; [58] recruits ICA for solving the offline scheduling problem with rejection. [59] uses ICA for solving a bi-criteria scheduling of the assembly flowshop problem. [60] employed ICA to solve the product mix-outsourcing problem. Besides, [61] has applied ICA to the schedule of receiving and shipping trucks in cross-docking systems. Further, [62] uses this algorithm for balancing of Stochastic U-type Assembly lines. The Dynamic Cell Formation Problem with Production Planning is also solved via ICA [63]. In [64], ICA is applied to Optimize the Huge-Combinatorial problem.

However, to the best of our knowledge, there have not been any applications of this novel algorithm, *ICA*, in financial management/financial engineering problems and particularly to the portfolio selection problem. In this paper, an ICA-based approach, which maps the portfolio problem to the ICA algorithm, is proposed for the portfolio selection with *simultaneous risk minimization/return maximization*, as the objective. Stock weights, are mapped to/represented by ICA countries and manipulated to achieve the optimum answer.

The rest of the paper is organized as follows: In section 2, a brief review of MPT's literature is given. Section 3 is devoted to investigating the investment selection process. In Section 4, a brief discussion about function/portfolio optimization is provided. Section 5 describes the methodology of this research and presents the proposed ICA-based approach for portfolio selection, besides, the data/population for validating the model will be discussed there. In the next section, the proposed ICA-based approach is validated and implemented using real market data; *i.e.* a portfolio is optimized via the proposed ICA approach using real market data, compared with *the market portfolio* and finally, the findings are discussed, section 6. Eventually, the paper is concluded in section 7, which summarizes the paper and talks about the conclusions and future works.

Literature Review

Markowitz Model: Modern Portfolio Theory (MPT): The insight for which Harry Markowitz received the Nobel Prize was first published in 1952 in an article entitled *Portfolio Selection* [5] and more extensively in his 1959 book, *Portfolio Selection: Efficient Diversification of Investments* [65]. When published in 1952, Markowitz's ideas scarcely took the investment profession by storm. As with insights from other researchers, his equation-filled presentation is over the heads of most investors [6]. Markowitz pointed out that the goal of portfolio management is *not* solely to maximize the expected rate of return, but instead to maximize *"expected utility"* - setting complexities aside, *"utility"* can be viewed as being synonymous with *"satisfaction"*. Markowitz began with the valid premise that all investors want a combination of high returns and low risk. That is, rational investors maximize their utility by seeking either;

- The highest available rate of return for a given level of risk. Or
- The lowest level of available risk for a given rate of return [1].

One of Markowitz's most important innovations was using the variance (or its square root, the standard deviation) of a distribution of likely returns (possible expected returns) as a measure of Risk. With his insight, Markowitz reduced the complicated and multidimensional problem of portfolio construction with respect to a large number of different assets, all with varying properties, to a conceptually simple two-dimensional problem known as *"mean-variance"* analysis [6].

The calculation of the expected return for an aggregate of portfolio of securities is relatively easy; it is merely the weighted average of the expected return of the individual securities. Nothing else is relevant:

$$\text{E (Rportfolio)} = \sum_{i=1}^{n} E(R_i)W_i \tag{1}$$

Where:

$E(R_{portfolio})$ = Portfolio's expected rate of return,
$E(R_i)$ = Security i's expected rate of return,
And W_i = The proportion of the portfolio's value invested in security *i*. [5]

The calculation of the combined variance is more complicated. The point to be considered is that the risk of a portfolio is *not* typically equal to the weighted average of the risks of its component securities. The risk of a portfolio depends not only on the risks of its securities, considered in isolation, but also on the extent to which they are affected similarly by underlying events [66]. The mentioned fact, similar affection by underlying events, actually refers to *covariance*. More elaboration of this topic is originated in the definition of *the correlation coefficients*. Correlation coefficient is the tendency between two variables to *"go together"*. Armed with mathematical measures, the relationship between the risk of a portfolio, consisting of *n* securities and the relevant variables could be drawn as (2):

$$VAR\left(R_{portfolio}\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_i \, Cov\left(R_i, R_i\right) \tag{2}$$

And substitution of parameters from the relationship that exists between the correlation coefficient and the covariance yields to (3):

$$VAR\left(R_{portfolio}\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_i \, S_{Ri} S_{Rj} r_{ij} \tag{3}$$

And therefore; portfolio's risk is extracted using 4:

$$Risk\left(R_{portfolio}\right) = \sqrt{VAR\left(R_{portfolio}\right)} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_i \, S_{Ri} S_{Rj} r_{ij}} \tag{4}$$

Where:

Risk ($R_{portfolio}$) = Risk of portfolio's rate of return,
VAR ($R_{portfolio}$) = Variance of portfolio's rate of return,
VAR (R_i) = Variance of security *i*'s rate of return,
COV (R_i,R_j) = Covariance between securities *i,j*'s rate of return,
n = the number of securities,
W_i or W_j = The proportion of the portfolio's value invested in security *i* or *j*,
r_{ij} = Coefficient of correlation between securities *i* and *j*,
S_i = Standard deviation for security *i*,
And S_j = Standard deviation for security *j* [5].

Investigating risk equation leads to some interesting results: a. In portfolios with perfectly positively correlated returns, diversification does not help. In such cases, diversification does not provide risk reduction, only risk averaging. b. In portfolios with perfectly negatively correlated returns, diversification can eliminate the risk. This principle motivates all *hedging strategies*. c. When the returns are uncorrelated, diversification indeed helps and provides substantial risk reduction; it provides the basis for insurance, or *risk pooling*. At first glance it might appear that diversification has no effect here. However, this is not at all the case. It can be shown that the risk of the portfolio is less than the risk of its component securities [66]. For example, imagine a portfolio of equal parts of *N* securities, each with an equal risk of S%. Then by using (4) and simplifying, (5) is yielded:

$$S_{portfolio} = \frac{S\%}{\sqrt{N}} \tag{5}$$

Using limit theorem and finding the limit of (5) as $N \to \infty$, (6) is achieved:

$$\lim N \to \infty^{S} S_{portfolio} = \frac{S\%}{\sqrt{N}}$$ (Source: Calculated by Authors) (6)

Where in Both Equations Above:

$S_{portfolio}$ = Standard deviation of the portfolio, a measure of portfolio's risk.
S% = Standard deviation of each security, a measure of security's risk.
N = Number of securities in a portfolio.

Basic Assumptions of Markowitz Model:

- The investor does (or should) maximize the discounted (or capitalized) value of future returns. [5]; [67]
- Investors behave rational in investment decisions, which means they choose to hold efficient portfolios- portfolios, which maximize each investor's utility. [5].

Investment Selection Process via MPT: Each investor has specific indifference curves for his/her risk/expected-return preferences. A set of these curves is named investor's indifference map- an investor's willingness to trade off changes in risk against changes in expected return. In an efficient market, there exists an efficient set of investment alternatives, which is called efficient frontier in Markowitz words and financial jargon-a portfolio with minimum risk for a given level of expected return or maximum expected return for a given degree of risk. This efficient frontier can be plotted using Markowitz model [68]. The matching of the available investment alternatives along the efficient frontier, with the investor's highest indifference curve is the final step in the investment selection process; Fig. 1 illustrates the three mentioned steps.

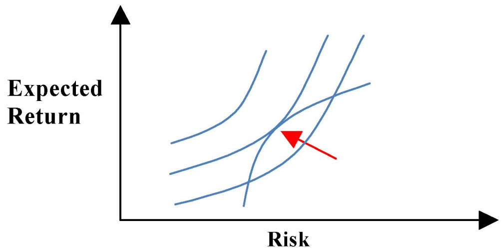
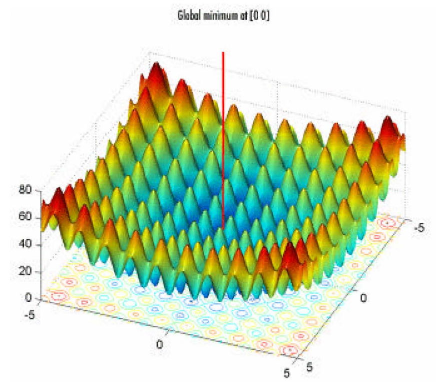


Fig. 1: Three steps of Investment selection process via MPT: Leading to the Optimal Portfolio by Matching of the efficient frontier with the investor's highest indifference curve



Source: Rastrigin's two-dimensional Function
Fig. 2: The concept of local optima and the global optimum

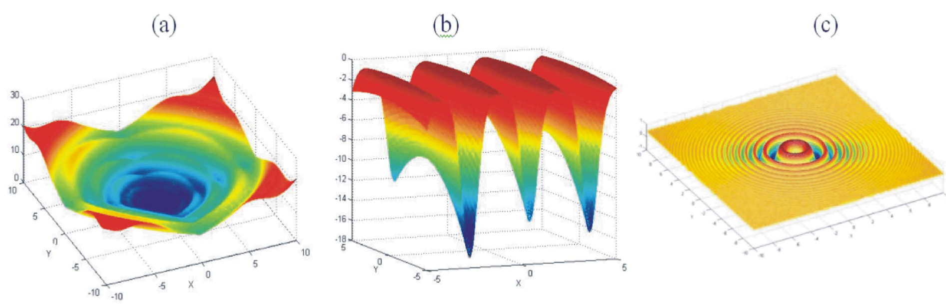


Fig. 3: Plots of some sample complicated functions for optimization

Function Optimization Methods: From a general point of view, optimization is the process of making something better [69]. Having a function *f(x)*, the aim of optimization is to find an argument *x*, whose relevant cost is optimum (usually minimum). Technically, Function optimization is the process of finding an optimal solution to an objective function describing a problem. Optimization can be either a minimization or maximization task. Optimization methods are divided into two major groups, classic optimizers, also known as gradient-based methods and (Meta) heuristic or evolutionary optimizers.

The problem with classic methods is that they can usually just find the local answers and optima, in other words, in a complicated problem, problems that are characterized by discontinuities, lack of derivative information, noisy function values, disjoint search spaces [70, 71] and nonlinear complicated equations, like that of Markowitz, classic solvers often stop solving as soon as reaching a local optimum.

Evolutionary algorithms have been designed primarily to address problems that cannot be tackled through traditional optimization algorithms. Although still there is no guarantee, heuristic methods usually turn out to achieve better results and better performances in contrast to their classic counterparts (An example of evolutionary method's better performance in portfolio problem could be found in [7]). This better performance goes back to their nature of design; they have been designed and created to *"jump out"* of *local optima* to reach *the global one*. They are supposed not to *"get* *stuck"* in local optima. In other words, because heuristic algorithms perform a wide random search, the chance of being trapped in local optima is deeply decreased. Fig. 2 illustrates the concept of local optima and the global optimum (*minimum* optima/optimum in this example). Besides, Fig. 3 shows some plots of complicated functions for optimization.

In recent years, heuristic function optimization has received extensive research attention. Several machine learning techniques such as neural networks [72, 73], evolutionary algorithms [74-76] and swarm intelligence-based algorithms [77,78,71], have been developed and applied successfully to solve a wide range of complex academic and practical optimization problems. Population-based search methods, a sub-group of heuristic optimizers, present a real and viable alternative to existing numerical optimization techniques. Population based optimization techniques can rapidly search large and convoluted search spaces and are not susceptible to suboptimal solutions. The Imperialist Competitive Algorithm (ICA) is the latest member of the family.

Portfolio Optimization: Markowitz demonstrated that once prepared, the foregoing security descriptions could be manipulated by portfolio optimization programs to produce an explicit definition of the efficient portfolio in terms of:

- The securities to be held.
- The proportion of available funds to be allocated to each. [1]

Methodology and Approach Validation

Methodology; ICA-Based Approach Introduction: This study aims at developing an alternative and efficient optimization methodology for resolving the portfolio selection problem and opening the application of Imperialist Competitive Algorithm to the issues for portfolio management. Hence, the *ICA-based* approach, to resolve the portfolio selection problem with the objective of simultaneous risk minimization/return maximization, is introduced in this paper. During last two decades, different heuristic methods have been applied to the portfolio selection problem, however, to the best of our knowledge, the ICA-based approach, which is quite a novel algorithm, is not applied to this problem yet. Based on this novelty and algorithm's strength in optimization problems (the next section illustrates this strength), ICA is chosen for representation in this paper. The proposed ICA algorithm is implemented in MATLAB [79] Software.

What is Imperialist Competitive Algorithm and why ICA for portfolio optimization?

Imperialist Competitive Algorithm (ICA) is a new socio-politically motivated global search strategy for dealing with different optimization tasks, which has recently been introduced by [38]. This evolutionary optimization strategy has shown great performance in both convergence rate and global optima achievement, though the applications were limited. A number of advantages with respect to other algorithms make ICA an ideal candidate to be used in optimization tasks. The algorithm has shown to be efficient, robust and flexible; within difficult (nonlinear, non-differentiable, or multi-modal) search spaces. In other words, it is well suited to handle nonlinear and non-convex design spaces and this derivative-free method can handle convoluted areas of search space with ease. ICA is computationally inexpensive in terms of both memory requirements and CPU speed [38,80,54,43,56].

All the mentioned facts make ICA a marvelous novel candidate for portfolio selection. Moreover, to the best of our knowledge, there have not been any applications of ICA in financial management/financial engineering problems and particularly, to the portfolio selection problem. Thus, in this paper, an ICA-based approach, which maps the portfolio problem to ICA algorithm, is proposed for the portfolio selection with simultaneous risk minimization/return maximization, as the objective. Stock weights, are mapped to/represented by ICA countries and manipulated to achieve the optimum answer.

Overview of Imperialist Competitive Algorithm: Similar to other evolutionary algorithms, this algorithm starts with an initial population. Each individual of the population is called a country. Some of the best countries (in optimization terminology, countries with the least cost) are selected to be the imperialist states and the rest, constitute the colonies of these imperialists, in other words, this algorithm maintains a set of candidate solutions, referred to as country/imperialist - some of the best countries constitute the imperialists. From the point of view of optimization categories, ICA is a *population based optimization tool*, where the system is initialized with a population of *random* countries and the algorithm searches for optima by updating empires via Imperialist *competition*. The performance of each country is measured using a predefined fitness function which encapsulates the characteristics of the optimization problem; *i.e.* the fitness of each country can be evaluated according to the objective function of the optimization problem. In a more detailed presentation: all the colonies of initial countries are divided among the mentioned imperialists based on the imperialist's power. The power of each country - the counterpart of fitness value in the GA- is inversely proportional to its cost. The imperialist states, together with their colonies constitute some empires. After forming initial empires, the colonies in each of them start moving toward their relevant imperialist country. This movement is a simple model of assimilation policy, which was pursued by some of the imperialist states historically. The total power of an empire depends on both the power of the imperialist country and the power of its colonies. This fact is modelled by defining the total power of an empire as the power of the imperialist country plus a percentage of mean power of its colonies.

The search procedure of a population-based algorithm such as ICA consists of two main phases, exploration and exploitation. The former is responsible for the detection of the most promising regions in the search space, while the latter promotes convergence of the countries towards the best detected solution. These two phases can take place either once or successively during the execution of the algorithm. Proper selection of internal parameters, affects ICA's trade-off between exploration and exploitation, albeit there is no formal procedure to determine the optimal quantity of parameters. Therefore, some rules of thumb, trial and error and the experience of ICA user are related in determining the internal parameters.

Creation of Initial Empires: The goal of optimization is to find an optimal solution in terms of the variables of the problem. Algorithm-user forms an array of variable values to be optimized. In the GA terminology, this array is called *"chromosome"*, in ICA jargon the term *"country"* is used for this array. In an N_{var}- dimensional optimization problem, a country is a $1\times N_{var}$ array. This array is defined as (7):

$$country=[p_1, p_2, p_3, ..., p_{N_{var}}] \tag{7}$$

Where:

- p_is are the variables to be optimized.

The variable values in the country are represented as floating point numbers. Each variable in the country can be interpreted as a socio-political characteristic of a country. From this point of view, the algorithm searches for the best country which is the country with the best combination of socio-political characteristics such as *culture, language, economical policy, ...* . From optimization perspective, this leads to finding the optimal solution of the problem, the solution with the least cost value. Fig. 4 shows the interpretation of a country using some socio-political characteristics.

The cost of a country is found by calculation of the cost function f at variables (p_1, p_2 p_3,....p_{Nvar}) which is defined by (8):

$$cost= f(country)=f(p_1, p_2, p_3, ..., p_{N_{var}}) \tag{8}$$

To begin the optimization, initial countries of size $N_{country}$ are produced, then, N_{imp} of the most powerful countries are selected as imperialists to constitute the empires. The remaining N_{col} of the initial countries will be the colonies, each of which belongs to an empire ($N_{Country}$, N_{imp} and N_{col} are predefined internal parameters of the algorithm already defined by the user). To constitute the initial empires, the colonies are divided among imperialists based on the imperialist's power; that is, the initial number of colonies of an empire should be directly proportionate to its power. To proportionally divide the colonies among imperialists, the normalized cost of an imperialist is calculated using (9):

$$C_n = c_n - \max_i\{c_i\} \tag{9}$$

Where:

- C_n is its normalized cost.
- *And* c_n is the cost of the n^{th} imperialist,

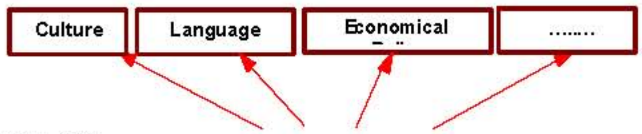
Having achieved the normalized cost of all imperialists, the probability of gaining countries for each of the imperialists could be defined as (10):

$$p_n = \left| \frac{C_n}{\sum_{i=1}^{N_{imp}} C_i} \right| \tag{10}$$

Where:

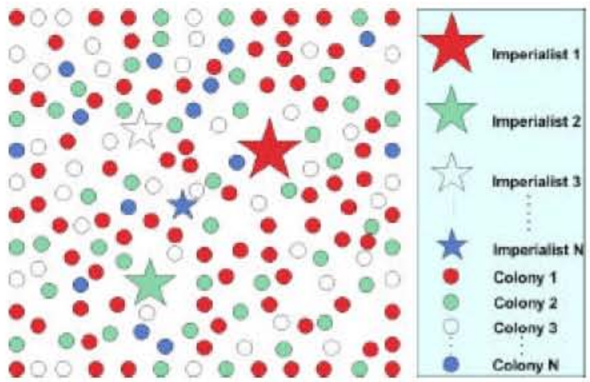
- P_n is the initial power of gaining countries for the n^{th} imperialist,
- C_n is its normalized cost,
- *And* C_i is the cost of the i^{th} imperialist.

The initial colonies are divided among empires based on their power. Thus, the initial number of colonies of the n^{th} empire follows (11):



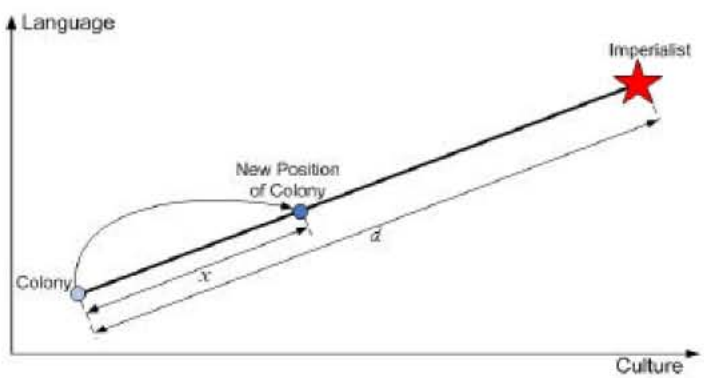
Source: [38]

Fig. 4: The candidate solutions of the problem, called country, consisting of a combination of some socio-political characteristics such as culture, language and economical policy, in ICA.



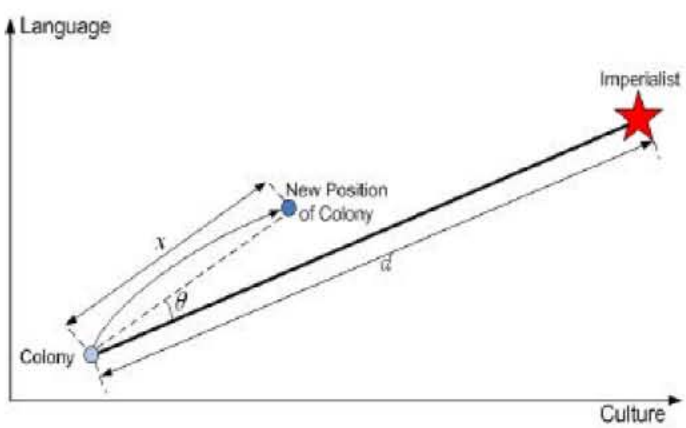
Source: [38]

Fig. 5: Generating the initial empires: The stronger an imperialist is (darker and bigger ★ marks in this figure), the more colonies it possesses (●/○ marks in this figure)



Source: [38]

Fig. 6: Movement of colonies toward their relevant imperialist



Source: [38]

Fig.7: Movement of colonies toward their relevant imperialist in a randomly deviated direction

$$N.C._n = round\{p_n.N_{col}\} \quad (11)$$

Where:

- $N.C._n$ is the initial number of colonies of the n^{th} empire,
- P_n is the initial power of gaining countries for the n^{th} imperialist,
- *And* N_{col} is the total number of initial colonies.

To divide the colonies, $N.C._n$ of the colonies are randomly chosen and given to the n^{th} imperialist. These colonies, along with the n^{th} imperialist constitute the n^{th} empire. Fig. 5 shows the initial empires along with countries. As shown in this figure, stronger empires - those with less cost- have a greater number of colonies, while weaker ones have less. In this figure, imperialist 1, the biggest and darkest star, has formed the most powerful empire and consequently has the greatest number of colonies; colony 1 which is illustrated by the darkest circles.

Assimilation

Movement of Colonies Toward Their Relevant Imperialist: Pursuing assimilation policy, the imperialist states tried to absorb their colonies and make them a part of themselves, historically. More precisely, the imperialist states made their colonies move toward themselves along different socio-political axes such as culture, language and economical policies. In the ICA, this process is modelled by moving all colonies toward their relevant imperialist and along different optimization axes, Fig. 6 illustrates this movement. Considering a 2-dimensional optimization problem in this figure, the colony is absorbed by the imperialist in the *culture* and *language* axes. Thereafter, the colony will get closer to the imperialist in these axes. Continuation of assimilation will cause all the colonies to be fully assimilated into the imperialist.

In a detailed model of assimilation, a colony moves toward the imperialist by x units, fig. 6. The new position of the colony is shown in a darker color. The direction of the movement is the vector from the colony to the imperialist. In this figure, x is a random variable with uniform (or any proper) distribution. Then, the related equation is (12):

$$x \sim U(0, \beta \times d) \quad (12)$$

Where:

β is a number greater than 1; $\beta > 1$ causes the colonies to get closer to the imperialist state from both sides. And d is the distance between the colony and the imperialist state.

Historically, assimilating the colonies by the imperialist states did not result in direct movement of the colonies toward the imperialist. That is, the direction of the movement does *not* necessarily exactly coincide with the vector from the colony to the imperialist; there usually is a bit deviation. To model this fact and more importantly to increase the ability of searching more area around the imperialist, *i.e.* to increase exploration of the algorithm, a random amount of deviation is added to the direction of movement. Fig. 7 shows this modelling. In this figure, è is a parameter with uniform (or any proper) distribution. Therefore, we will have (13):

$$? \sim U(-?, ?) \quad (13)$$

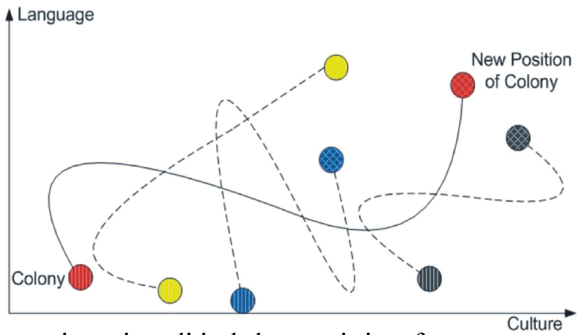
Where:

γ is a parameter that adjusts the deviation from the original direction.

Although the values of β and γ are arbitrary, in most of the implementations, a value of about 2 for β and about $\pi/4$ (Rad) for γ results in good convergence of countries to the global optimum.

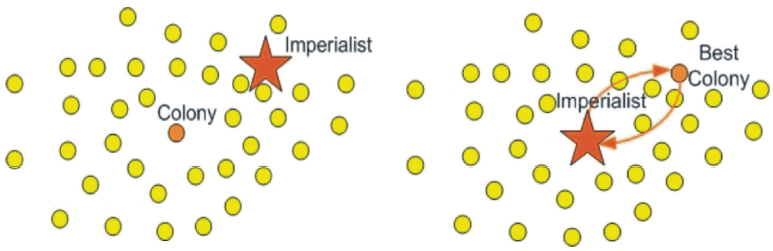
Revolution

A Sudden Change in Socio-political Characteristics of a Country: Revolution is a fundamental change in power or organizational structures of countries/empires that takes place in a relatively short period of time. In the ICA terminology, revolution causes a country to change its socio-political characteristics suddenly; that is, instead of being assimilated by an imperialist, the colony randomly changes its position in the socio-political axes. Fig. 8 illustrates the revolution in Culture-Language axes. Efficient rates of revolution, increases the exploration of the algorithm and prevents the early convergence of countries to local optima (minima). The revolution rate in the algorithm parameters' section indicates the percentage of colonies, which will randomly change their positions. Very large values of revolution rate decrease the exploitation/increase the exploration power of the algorithm and may reduce its convergence rate, while very small values, concentrate the algorithm to nearby regions of the design space and increase the exploitation/decrease the exploration power of the algorithm; probably leading to early convergence and prematurity. Therefore, large and small values of revolution rate cause a trade-off between exploration and exploitation to exist and should be taken care of with care. In the simulations of this paper,



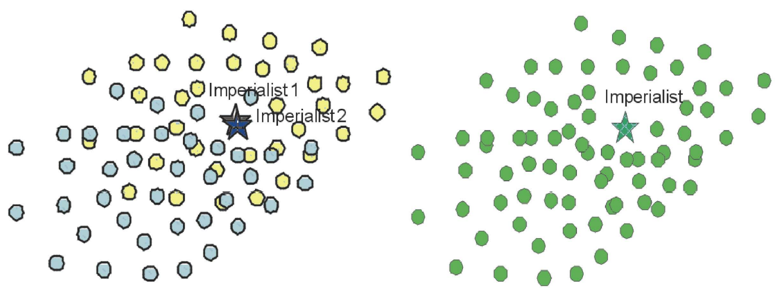
Source: [38]

Fig. 8: Revolution: a sudden change in socio-political characteristics of a country



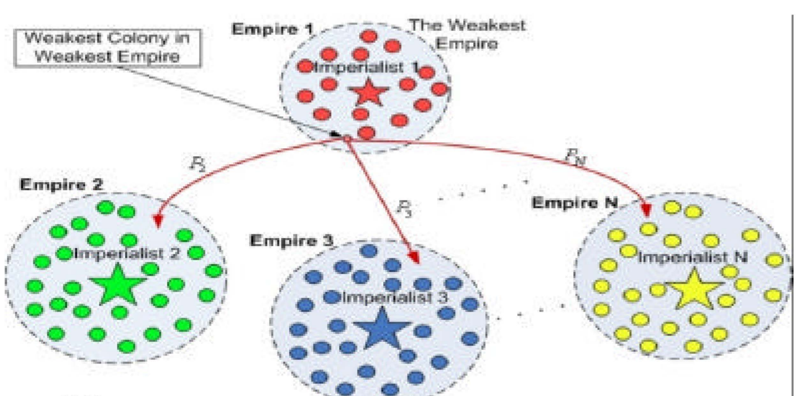
Source: [38] A Source: [38] B

Fig. 9(a): Exchanging the positions of a colony and the imperialist Fig. 9(b). The entire empire after the position exchange



Source: [38] A Source: [38] B

Fig. 10(a): The Empires before Uniting Fig. 10(b). The resultant Empire: The empire resulting from uniting two empires



Source: [38]

Fig.11: Imperialistic competition: The more powerful an empire is, the more likely it is to possess the weakest colony of the weakest empire.

the revolution rate is 0.3 and found consistent with the problem; that is, 30 percent of colonies in the empires change their positions randomly.

Exchanging Positions of the Imperialist and a Colony: In the movements toward the imperialist, a colony might reach a position with lower cost than its relevant imperialist. In this case, the position of the imperialist and the colony is changed. Thereafter, the algorithm goes on considering the new imperialist with new position and assimilating the colonies toward this new position. Fig. 9(a) depicts the position exchange between a colony and the imperialist. In this figure, the best colony of the empire is shown in a darker color. This colony has a lower cost than its imperialist. Fig. 9(b) shows the empire after exchanging the position of the imperialist and the colony.

Uniting Similar Empires: In the movement of colonies and imperialists toward the global optimum of the problem, some imperialists might move to similar positions. If the distance between two imperialists becomes less than the *threshold distance*, they both will constitute a new empire which is a combination of these empires. All the colonies of two empires become the colonies of the new empire and the new imperialist will be in the position of one of the two imperialists, usually the one with lower cost. Fig.s 10(a) and 10(b) show the uniting process of two empires before and after uniting, respectively.

Total Power of an Empire: Total power of an empire is mainly affected by the power of the imperialist country. However, the power of the colonies of the empire affects the total power of that empire, albeit negligible. This fact is modelled by defining the total cost of an empire using (14):

$$T.C. = Cost(imprialist_n) + \xi mean\{Cost(colonies\ of\ empire_n)\} \tag{14}$$

Where:

- $T.C._n$ is the total cost of the n^{th} empire,
- ξ is a positive small number; small values of ξ cause the total power of the empire be determined by just the imperialist, its larger amounts lead to an increase in the role of colonies in determining the total power of an empire. In most of the implementations, the value of 0.1 for ξ leads to good results.

Imperialist Competition: All empires try to take the possession and control of colonies that belong to other empires. The imperialistic competition gradually brings about a decrease in the power of weaker empires and an increase in the power of more powerful ones. The imperialistic competition is modelled by picking some (usually one) of the weakest colonies of the weakest empire and making a competition among all empires for possessing these (this) colonies. Fig. 11 shows a big picture of the modelled imperialistic competition. In this competition, based on its total power, each of empires has a *likelihood* of taking possession of the mentioned colonies. In other words, these colonies will not definitely be possessed by the most powerful empires; rather, these empires are more likely to possess the colonies.

To begin the competition, first a colony of the weakest empire is chosen. Then, the possession probability of each empire is calculated using the possession probability equation; P_P is proportionate to the total power of the empire. The normalized total cost of an empire is simply obtained via (15).

$$N.T.C._n = T.C._n - \max_i \{T.C._i\} \tag{15}$$

Where:

- $N.T.C_n$ is the normalized total cost of the n^{th} empire,
- *And* $T.C._n$ is the total cost of the n^{th} empire.
- Having found the normalized total cost, the possession probability of each empire could be achieved using (16):

$$p_{p_n} = \left| \frac{N.T.C._n}{\sum_{i=1}^{N_{imp}} N.T.C._i} \right| \tag{16}$$

Where:

- P_P the possession probability of each empire for possessing colonies,
- *And* $N.T.C_n$ is the normalized total cost of the n^{th} empire.
- To divide the mentioned colonies among empires, vector P is formed as follows in (17):

$$\mathbf{P} = \left[p_{p_1}, p_{p_2}, p_{p_3}, \ldots, p_{p_{N_{imp}}} \right] \tag{17}$$

1) Select some random points on the function and initialize the empires.
2) Move the colonies toward their relevant imperialist (Assimilation).
3) Randomly change the position of some colonies (Revolution).
4) If there is a colony in an empire which has lower cost than its relevant imperialist, exchange the positions of that colony and the imperialist.
5) Unite the similar empires.
6) Compute the total cost of all empires.
7) Pick the weakest colony (colonies) from the weakest empires and give it (them) to one (some) of the empires considering probabilities (Imperialistic competition).
8) Eliminate the powerless empires.
9) If the stopping criterion is satisfied, then stop, if not, go to step 2.

Source: [38]

Fig.12: Pseudo code of the Imperialistic Competitive Algorithm

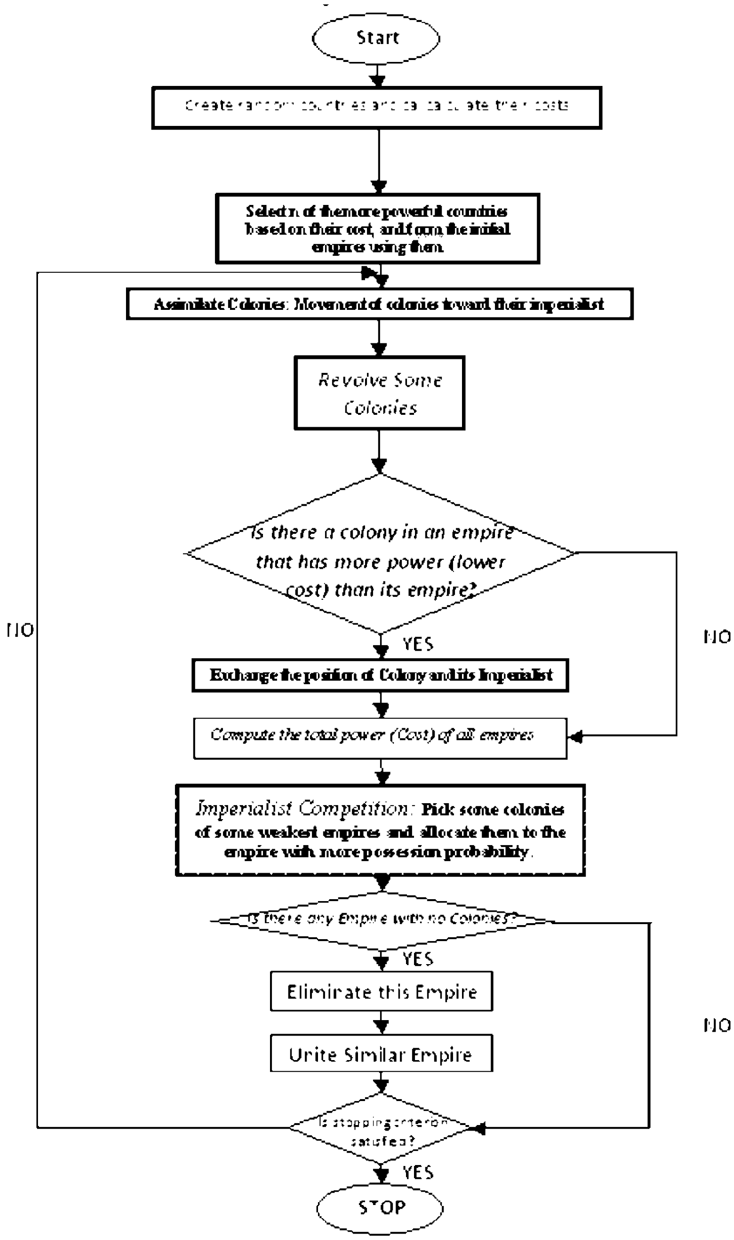


Fig.13: ICA Flowchart

Then, the vector R, whose elements are uniformly distributed random numbers - and has the same size as P - is created via (18):

$$R=\left[r_1, r_2, r_3, \ldots, r_{N_{imp}}\right] \quad r_1, r_2, r_3, \ldots, r_{N_{imp}} \sim U(0,1) \tag{18}$$

By Subtracting R from P, vector D is formed in (19):

$$\begin{aligned}\mathbf{D}=\mathbf{P}-\mathbf{R}&=\left[D_1, D_2, D_3, \ldots, D_{N_{imp}}\right]\\ &=\left[p_{p_1}-r_1,\ p_{p_2}-r_2,\ p_{p_3}-r_3, \ldots, p_{p_{N_{imp}}}-r_{N_{imp}}\right]\end{aligned} \tag{19}$$

Using vector D as a reference vector, *the so-called weak colony (colonies)* is handed to the empire whose relevant index in D vector is maximum.

The process of selecting an empire is similar to the roulette wheel process in GA, used in selecting parents; however, this method of selection is much faster than the conventional roulette wheel because it is not required to calculate the cumulative distribution function and the selection is based on only the values of probabilities. Hence, the process of selecting the empires can solely substitute the roulette wheel in GA and increase its execution speed, as well.

Algorithmic Description: Based on the above explanations, the main steps of ICA are summarized in the *pseudo-code* of fig. 12. The continuation of the mentioned steps will hopefully lead the countries converge to the global optimum of the cost function (by default, the global minimum). Different criteria could be used as stopping criteria of the algorithm. One idea is to use a number of maximum iterations of the algorithm, called maximum decades. Another could be the end of imperialistic competition as a stopping point; *i.e.* when there is only one empire left, it could be considered as the stopping criterion of the ICA. Furthermore, the optimization task could be stopped when the algorithm's best solution cannot be improved for some consecutive decades. It can be noticed that ICA shares many common points with GA, PSO and other population-based methods. Some of these common points could be the random generation of initial population, search for optima by updating generations or iterations and evaluation of a fitness or objective for possible solutions. However, they have different sharing mechanisms for optimization during the search. Fig. 13 illustrates ICA flowchart.

ICA Country Representation for the Portfolio Selection Problem: In order to apply ICA to the portfolio problem, it is necessary to find a suitable mapping between the stock portfolio and the ICA country. Markowitz risk-return equations along with portfolio weights as countries, is used to bridge the ICA country with the stock portfolio selection. In this case, through ICA countries, optimal portfolio weights can be searched from a population of country-represented weights that are updated according to ICA mechanism. Moreover, instead of obtaining initial portfolio weights from different classic or heuristic rules or methods, the initial weights represented by ICA country are randomly generated.

As a point in an *n-dimensional* space, the *n* elements of an ICA country can stand for the *n* potential stocks in a portfolio under construction. Hence, the *n* parameters of a country's position can represent the weights of the *n* stocks in the would-be portfolio.

Based on the country representation of the portfolio and in order to apply the constraints concerned with the portfolio problem[3], the ICA-based methodology for the portfolio selection problem is developed and implemented in this research.

According to [3] risk of a *carefully selected and diversified* portfolio with at least 15 stocks, equals the risk of a portfolio of the whole market, known as the *market portfolio*. Therefore, in order to have at least 15 stocks in the constructed portfolio, the following constraint, (20), is to be considered:

- Upper Bound of portfolio's value invested in security i =

$$\frac{\textit{Total Budjet of Portfolio\%}}{\substack{\textit{The least number of stocks in the portfolio to}\\ \textit{have a risk close enough to the market portflio}}}=\frac{100\%}{15}=6.66\% \tag{20}$$

Besides, the lower limit for a stock weight in the portfolio is supposed not to be negative (Borrowing and lending are not assumed allowed). Thus, because the ICA country represents a series of stocks' weights in the

[6] http://www.irbourse.com

[7]Available at the library of Tehran Stock Exchange Central Organization.

[8] In order to save space and not to lengthen the paper, the details of the programming and the related M-files and codes, prepared and written by authors, are not provided in the paper text. However, they could be available to any reader by sending a request e-mail to the corresponding author.

portfolio that range from 0 (0%) to 0.0666 (6.66%), all parameters of the *n-dimensional* country positions, either initialized or updated during the search, must be limited to [0, 0.0666] or [X^{min}, X^{max}], avoiding infeasible country positions that can lead to slow ICA search. Each parameter of the initialized or updated position that is beyond [X^{min}, X^{max}] can be adjusted in the following form, (21):

$$\text{if } x_{ij}(t) > X^{max} \text{ then } x_{ij}(t) = X^{max}$$
$$\text{else if } x_{ij}(t) < X^{min} \text{ then } x_{ij}(t) = X^{min}. \qquad (21)$$

Where:

- $x_{ij}(t)$ represent the j^{th} element of the i^{th} country,
- X^{min} and X^{max} are lower and upper bounds of stocks, respectively.

The third constraint that is to be considered is a *linear constraint*; this linear constraint requires the sum of countries be set to 1. The reason to this constraint goes back to the nature of lending and borrowing possibilities, which are assumed forbidden in here. In other words, spending any amount more or less than the pre-determined budget for the portfolio is not allowed. Lending (or spending less than the pre-determined portfolio budget) causes the budget become less than 100% or 1 and on the other hand, borrowing (or spending more than the pre-determined portfolio budget) leads to more than 100% budgets, which are not possible alternatives in this research and also usually in real situations. Hence, to satisfy this constraint, the sum of countries is set to 1 (or 100% of the budget).

Calibration of ICA's Internal Parameters: For the number of countries in the population, *i.e.*, M or called *P-size*, more countries may increase success in searching for optima because of sampling state space, more thoroughly. However, more countries require more evaluation runs, leading more optimization cost. Anyway, a rule of thumb says: *In population-based algorithms, like GA, PSO and ICA, it seems safe and conservative to choose the populations size equal to 10 times the number of variables!* Therefore, a P-size of at least 10 times the number of variables, stocks in the portfolio problem, is selected for the ICA-based approach.

Other ICA parameters are set via *trial and error* in successive iterations of the algorithm. Therefore, they will be revealed later on, in the findings section.

Validation of the ICA-Based Approach via an Experimental Test: The performance of the proposed ICA-based approach has been evaluated on a test bed of a real stock market via constructing a portfolio in this market and applying the constructed portfolio to real data of market for a six month period following portfolio's construction, known as *the test period in this research*. Thereafter, a comparison between ICA-based portfolio and *the market portfolio* is performed in order to evaluate the ICA's performance and also to determine its efficiency in portfolio construction.

Population and Data

Population: Testee Stock Market: This study concerns with portfolio selection in *Tehran stock exchange market* and from the stocks of top *50 listed companies* in that market. The list of these companies is extracted from the library of Tehran Stock Exchange Central Organization. It was the newest available updated list up to *January, 21st, 2010*. As one may ask, *"why top 50 companies?"* Here comes the explanation: the only way the systematic risk can be lowered is to expand the definition of the *"market"* to include dissimilar markets [1]. Internationally diversified portfolios are much less risky than the limited ones [81]; [82]. When we select our portfolios for research purposes and in limited markets such as the stock exchange market of only *one* zone, like that of Tehran, no international diversification is possible (and also affordable). Therefore, in such situations and in order to bear the least possible risk, it is conservative to select the portfolio from companies that are listed as top and successful companies and are probably outstanding in their own industry - at least most of the time. This way, the real performance of method could be revealed.

Samples and Historical Data: Table 1 shows the name, sector of activity and data for 50 companies' stocks average rate of return, during six periods[4], starting from *March, 20th, 2004* and ending in *March, 20th, 2009*. Each period is considered 1 total financial year. Besides, Fig. 14 illustrates these average rates of returns in a column diagram. Note that Iranian's New Year (*Norouz*) and so

[6] http://www.irbourse.com

[7] Available at the library of Tehran Stock Exchange Central Organization.

[8] In order to save space and not to lengthen the paper, the details of the programming and the related M-files and codes, prepared and written by authors, are not provided in the paper text. However, they could be available to any reader by sending a request e-mail to the corresponding author.

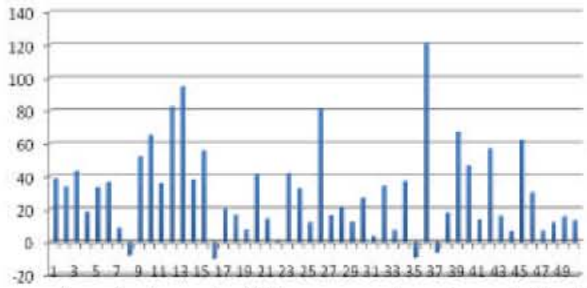


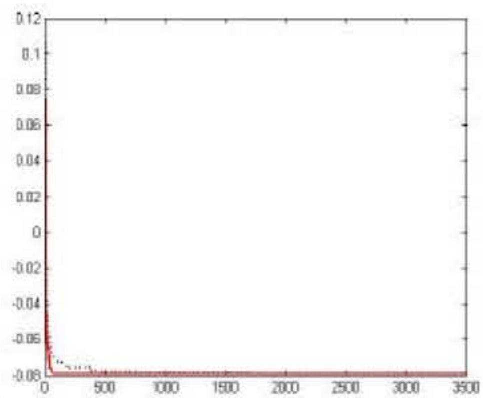
Fig. 14: Column diagram of average rates of returns for 50 top companies, six periods of Tehran Stock Market

Source: Tehran Stock exchange market official website

Fig. 15: Market Index, six periods of Tehran Stock Market

Source: Tehran Stock exchange market official website

Fig. 16: 50 top companies' Index, six periods of Tehran Stock Market



(Source: Calculated by Authors)

Fig. 17: Mean/Best fitness functions value versus the ICA algorithm's iteration numbers

---

[6] http://www.irbourse.com

[7] Available at the library of Tehran Stock Exchange Central Organization.

[8] In order to save space and not to lengthen the paper, the details of the programming and the related M-files and codes, prepared and written by authors, are not provided in the paper text. However, they could be available to any reader by sending a request e-mail to the corresponding author.

the financial year begins on *March, 21st*. Therefore, the end of a financial year in Iran is on *March, 20th* each year, which is important to be considered because of the effects of New Year on financial issues, the same as January effect (which happens in Iran's market as well and could be called Farvardin[5] effect.) *"Why Six periods?"* Six periods were selected for at least two reasons: 1. the most possible information could be extracted when considering six recent periods; any consideration more than six periods could damage the data, because some companies were not present in the market in the preceding periods. 2. Fig.'s 15 and 16 show the Index diagram for Tehran stock market for total market and the 50 top companies, respectively. To extract a rigorous covariance between pairs of stocks - needed for portfolio selection - both prosperity and depression economic periods, should be considered. Considering this fact, a period of at least six years seems conservative. Moreover, to achieve an exact covariance matrix, monthly data seems more conservative; thus, monthly return data of 50 top companies and for *seventy-one* months are selected for this research. In other words and to be exact, seventy-one return data, starting from *March, 20th, 2004* and ending in *January, 20th, 2010* are selected as historical data for portfolio construction.

Test Period Data for Model Validation: In order to validate the performance and efficiency of the ICA-based selected portfolio, real data was gathered for a period of six months, since *January, 21st, 2010* up to *July, 21st, 2010*; the period was chosen as long as possible to omit any random effect that may exist in portfolio performance and also omission of Farvardin effect.

Data Gathering Tools and Assumptions: In order to perform this research, raw data was gathered using the following tools: Library studies of Tehran Stock Exchange market, Tehran Stock Exchange Corporation's Official websit[6] and databases of two special softwares designed for Tehran Stock Markets; Tadbirpardaz and Rahavard Novin[7].

Finally, let's refer to a common procedure which often happens in data gathering for portfolio selection, but rarely mentioned. Because historical data is readily at hand, it is usually used as representatives of future data. Cautions must be taken when using the data in this way, because in some cases, historical data may not be good and real representatives of future data. Portfolio equations need *ex-ante'* data, which are somehow different from historical data. Assuming historical data instead of *ex-ante'* data is an unspoken assumption that should be taken care of and used with cautious.

Findings and Discussion

Calculations and Results: Applying covariance equations to the raw data of rates of return, a covariance matrix between each pair of stocks is approximated; this matrix is 50×50 and we name it COV.matrix. Because the proposed ICA algorithm is implemented in MATLAB code and optimization methods that are programmed with the MATLAB use *minimization objective* as the default procedure, we had to code the portfolio optimization problem as *minimization of the fitness function*[8], in which, simultaneous risk minimization/return maximization is implemented via programming skills.

Using the ICA representation for portfolio selection, table 2 is yielded. Table 2 is the optimum portfolio selected via ICA algorithm using COV.matrix. Percentage numbers in table 2, *known as* W_i, are the proportion of the portfolio's value invested in security *i*. Fig. 17 illustrates the amounts of Mean/Best fitness functions versus the algorithm iteration numbers for the constructed portfolio. Test trials suggest the algorithm parameters represented in Table 3; *i.e.* table 3 acts out the best internal parameters of ICA approach found to be consistent with the portfolio selection problem, so that, next ICA user could invoke the parameters of this table as a good starting point and perhaps the best one. Substituting the weights of Table 2 in (1) yields to:

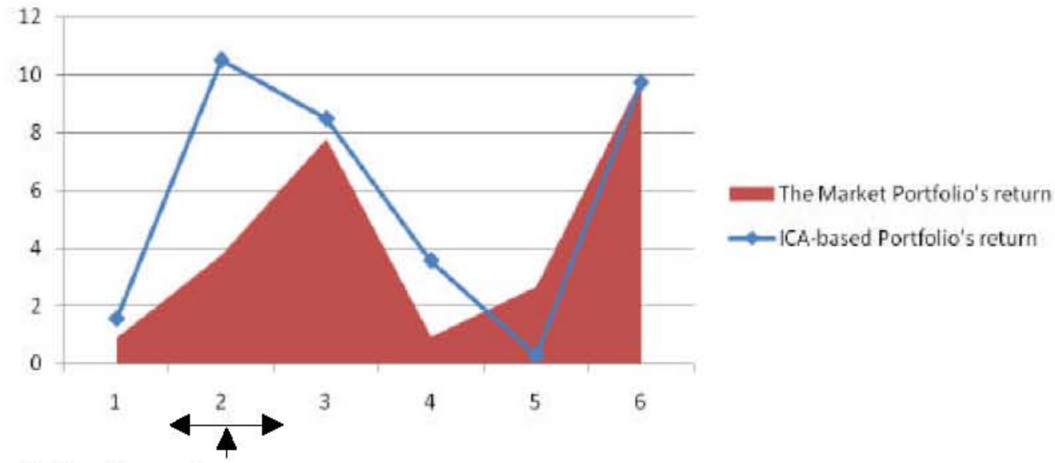
- Expected Return of the Portfolio= $E_{R\ \text{portfolio ICA-based constructed}}$= 0.359908 or 35.9908%

And the substitution in (3) leads to:

- VAR ($R_{R\ \text{portfolio ICA-based constructed}}$) = 0.0789or7.89%, so using (4) we'll have the following amounts:
- Portfolio's Risk = Risk ($R_{R\ \text{portfolio ICA-based constructed}}$) = 0.280899 or 28.0899%.

  Model Validation: Portfolio Testing Results

As already explained, the performance and efficiency of the proposed ICA-based approach is to be evaluated by applying the constructed portfolio to real data of market for a six month period after portfolio's construction and then, a comparison between the ICA portfolio and the market portfolio is essential. Therefore, the ICA portfolio is applied to the market data of proceeding months, hence; securities were selected and hold, accordingly. This holding was since *January, 21st, 2010* up to *July, 21st, 2010*; the period was chosen as long as possible to omit any random effect that may exist in portfolio performance and also omission of Farvardin effect. The return result of the portfolio is detailed in Table 4.



Source: Calculated by Authors

Fig. 18: ICA-based portfolio's return in contrast to the market portfolio's return; during 6 months called test period (Diagram Numbers are expressed as percentage)

Table 1: Average rates of returns for 50 top companies, six periods of Tehran Stock Market and columns of company sectors of activity (The numbers are expressed as the percentage)

Company Name	Average Return for Six Periods	Sector	Company Name	Average Return for Six Periods	Sector	Company Name	Average Return for Six Periods	Sector
1.Iran Khodro	38.41	Motor Vehicles and Auto Parts	18.Petro. Inv.	16.22	Chemicals and By-products	35.Azar Refract.	-9.49	Other nonmetallic Mineral Product
2. Iran Khodro D.	33.40	Motor Vehicles and Auto Parts	19. Iran Ind. Dev.	7.49	Investment Companies	36.Calcimine	121.19	Basic Metals
3. EN Bank	42.85	Banks, Credit and Other Financial Institutions	20. Rena Investment	41.28	Motor Vehicles And Auto Parts	37.Iran Carbon	-6.26	Chemicals and By-products
4.Pars Darou	18.12	Pharmaceuticals	21.Insurance Inv.	13.83	Investment Companies	38.Kaf	17.49	Chemicals and By-products
5.Abadan Petr.	32.92	Chemicals and By-products	22.Ind. and Mine Inv.	1.02	Investment Companies	39.Gazlouleh	66.53	Rubber and Plastic Products
6.Khark Petr.	35.91	Chemicals and By-products	23.Ghadir Inv.	41.68	Diversified Industrials (Conglumerates)	40.Bahman Group	46.18	Motor Vehicles And Auto Parts
7.Farabi Petr.	8.23	Chemicals and By-products	24.Behshahr Group	32.18	Investment Companies	41.Sadid Group	13.27	Basic Metals
8. Zangan Equip.	-8.31	Refined Petroleum Products and Nuclear Fuel	25.Housing Inv.	11.71	Real Estate And Construction	42.Looleh o Mashin Sazi	56.65	Basic Metals
9. Iran Tractor	51.49	Machinery and Equipment	26.Metals and Min.	81.13	Metal Ores Mining	43.Niromohareke M.	15.54	Machinery and Equipment
10. Chadormalu	64.97	Metal Ores Mining	27. Melli	15.88	Investment Companies	44.Mehvar Sazan	6.09	Motor Vehicles and Auto Parts
11.Jaber Hayan P.	35.36	Pharmaceuticals	28. Oil Ind. Inv.	20.58	Refined Petroleum Products and Nuclear Fuel	45.Iran Zinc Mines	61.78	Metal Ores Mining
12.Zamyad	81.99	Motor Vehicles and Auto Parts	29.Tehran Cement	11.98	Cement, Lime and Gypsum	46.Iran Mn. Mines	29.87	Metal Ores Mining
13.Saipa	94.42	Motor Vehicles and Auto Parts	30. Fars Cement	26.47	Cement, Lime and Gypsum	47.Mehrcam Pars	6.55	Motor Vehicles And Auto Parts
14.Saipa Diesel	37.71	Motor Vehicles and Auto Parts	31. Shahdiran Inc.	3.09	Non-sugar Products	48.Motogen	11.60	Electric Machinery and Apparatus
15.Pension Fund	55.10	Diversified Industrials (Conglumerates)	32.Behshahr Ind.	33.95	Non-sugar Products	49.Behran Oil	15.34	Refined Petroleum Products and Nuclear Fuel
16. Buali Inv.	-10.08	Investment Companies	33. SADRA (Sanati Daryayi)	6.86	Industrial And Engineering Project Management	50.Pars Oil	12.70	Refined Petroleum Products and Nuclear Fuel
17.Pars Tousheh	19.75	Machinery and Equipment	34. Doroud	36.68	Other Non-metallic Mineral Products			

Source: Calculated by Authors

Table 2: ICA's Optimum portfolio, the proportion of the portfolio's value invested in security *i*, selected via ICA algorithm using COV.matrix. (The numbers are expressed as the percentage)

Company Name	Percentage of portfolio Value invested in stock	Company Name	Percentage of portfolio Value invested in stock	Company Name	Percentage of portfolio Value invested in stock
1.Iran Khodro	0	18.Petro. Inv.	0	35.Azar Refract.	0
2. Iran Khodro D.	0	19. Iran Ind. Dev.	0	36.Calcimine	6.66
3. EN Bank	0	20. Rena Investment	0	37.Iran Carbon	0
4.Pars Darou	6.66	21.Insurance Inv.	0	38.Kaf	0
5.Abadan Petr.	0	22.Ind. and Mine Inv.	0	39.Gazlouleh	0
6.Khark Petr.	0	23.Ghadir Inv.	0	40.Bahman Group	0
7.Farabi Petr.	0	24.Behshahr Group	6.66	41.Sadid Group	0
8. Zangan Equip.	4.3	25.Housing Inv.	0	42.Looleh o Mashin Sazi	6.66
9. Iran Tractor	6.66	26.Metals and Min.	0	43.Niromohareke M.	0
10. Chadormalu	6.66	27. Melli	0	44.Mehvar Sazan	0
11.Jaber Hayan P.	0	28. Oil Ind. Inv.	0	45.Iran Zinc Mines	0
12.Zamyad	0	29.Tehran Cement	0	46.Iran Mn. Mines	6.66
13.Saipa	2.45	30. Fars Cement	0	47.Mehrcam Pars	6.66
14.Saipa Diesel	0	31. Shahdiran Inc.	0	48.Motogen	6.66
15.Pension Fund	0	32.Behshahr Ind.	6.66	49.Behran Oil	6.66
16. Buali Inv.	0	33. Sanati Daryayi	0	50.Pars Oil	6.66
17.Pars Tousheh	6.66	34. Doroud Farsit	6.66		

Table 3: The best internal parameters of ICA approach consistent with the portfolio selection problem

Number of Initial Countries	500	Damp Ratio	0.99
Number of Initial Imperialists (Empires)	10	Uniting Threshold	0.02
Revolution Rate	0.3	Number of generations (Decades)	3500
Assimilation Coefficient	2	Time Limit	Infinitive
Assimilation Angle Coefficient	0.5	Fitness Limit	- Infinitive
Coefficient of countries impact on total power of the empire; ?	0.02	Function Tolerance	10-6

Source: Calculated by Authors

Table 4: Summary of calculations and a comparison between ICA portfolio's return results with market portfolio's return for each month of the test period

Portfolio/Monthly Return	Bahman †	Esfand ‡	Farvardin §	Ordibehesht ¶	Khordad ±	Tir Θ	Average Return During Six Months
ICA-based Portfolio	1.57	10.47	8.48	3.58	0.29	9.73	5.69
The Market Portfolio	0.9	3.78	7.8	0.93	2.67	9.88	4.33

† The 11th month of the year in Iranian's Calendar. ¶ The 2nd month of the year in Iranian's Calendar.
‡ The 12th month of the year in Iranian's Calendar. ± The 3rd month of the year in Iranian's Calendar.
§ The 1st month of the year in Iranian's Calendar. Θ The 4th month of the year in Iranian's Calendar.

Source: Calculated by Authors

Table 5: ICA-based portfolio's specifications and a contrast to the average achieved amount in the test period and also the monthly average of the market portfolio

Portfolio construction method/ Specification	Number of securities included (Diversification)	ICA Convergence Speed (According to decades or generations)	ICA Convergence Speed (Timely)	Expected Annual Return (%)	Risk (%)	Expected Monthly Return (%)	Risk (%)	Average *monthly* rate of return in application (Test period) (%)	Market portfolio's average monthly return (Test period) (%)
ICA (ICA-based Portfolio)	16 (out of 50)	3000	5 minutes and 8 seconds	35.9908	28.0899	2.9993	2.3408	5.69	4.33

Source: Calculated by Authors

In financial research, market portfolio is an important benchmark portfolio. In other words, once a portfolio outperforms the market portfolio, it is known as an outstanding and successful portfolio; otherwise, it is not. Fig. 18 illustrates a comparison between ICA-based portfolio's return and Tehran market portfolio's return during the test period. As it is obvious, ICA-based portfolio outperforms the market portfolio for four months out of six, *i.e.*, by achieving extremely larger amounts of returns, ICA approach shows a better performance in contrast to the market portfolio. Table 4 also shows a comparison between ICA portfolio's return results and the market portfolio's return for each month.

Table 5 outlines the ICA-based portfolio's expected return/risk, number of securities selected and its convergence speed - both timely and according to the generations, besides, this table illustrates the comparison between the *average* achieved amount in the test period and also the *monthly average* of the market portfolio.

DISCUSSION

The proposed ICA algorithm is considered efficient for at least three main reasons: 1. it never misses the optimal solution, namely, the algorithm is run repeatedly (twenty times) and it never misses the optimal solution. 2. the time and decades for every iteration remain the same as the count proceeds. Moreover, both timely and decadal speeds are fast. 3. ICA portfolio outperforms the market portfolio most of the times, 67% of the times. On the whole, findings show that the proposed algorithm can consistently handle the practical portfolio selection problem with good efficiency. These findings are discussed thoroughly in the following paragraphs.

As the number of stocks increases, the search space would grow exponentially. This reflects the advantage of ICA, in contrast to conventional optimization techniques, in being able to locate the solution efficiently when the search space is complex and large. Fig. 17 shows this efficiency schematically, this figure presents the process ICA went through to reach the optimum answer. As a close look may reveal, ICA has located the optimal solution after about 3000 iterations, respectively, whose relevant time was approximately 5 minutes; this is a fast and suitable speed, *i.e.* this speed for locating the solutions in such a complicated and large search space shows that the convergence speed is reasonable and the algorithm has a fast performance. Note that as it was already mentioned, increase in the number of stocks, leads to a more difficult and complicated problem. Therefore, the convergence speed of an optimization algorithm *in complicated search spaces* is of great importance.

In a comparison between the ICA-based portfolio and the market portfolio, which is an important index, it should be mentioned that the ICA-based portfolio outperforms the market portfolio most of the time, 67% of the test period (see fig. 18). Hence, outperforming the *market portfolio*'s average using ICA-constructed portfolio in a relatively long period of six months is assumed a success. This relatively long period omits any random effects that may exist and performs this omission with holding *only* 16 securities in contrast to 50 securities of the market portfolio. Considering the calculation procedures, the nature of dominancy for ICA's portfolio goes back to its ability in locating the global optimum.

Summary, Conclusions and Future Works: Markowitz demonstrated that the two relevant characteristics of a portfolio are its expected return and some measure of its risk- operationally defined as the dispersion of possible returns around the expected return. Rational investors will choose to hold efficient portfolios. The identification of efficient portfolios would require information on each security's expected return, variance and covariance of returns. Finally, once prepared, the foregoing security descriptions could be manipulated by portfolio optimization programs to construct the optimal portfolio. Optimization methods are divided into two major groups, classics and heuristics (or evolutionary algorithms). Evolutionary algorithms have been designed primarily to address problems that cannot be tackled through traditional optimization algorithms, problems like the portfolio selection problem. Recently, an evolutionary optimization algorithm has been developed based on the simulation of *socio-political evolution processes*, *Imperialist Competitive Algorithm.* This evolutionary optimization strategy has shown great performance in both convergence rate and global optimum achievement.

The Imperialist Competitive Algorithm (ICA)-based approach to resolve the portfolio selection problem with the objective of simultaneous risk minimization/return maximization was introduced in this paper. Security weights in a portfolio are represented by countries and the countries are manipulated to primarily satisfy three constraints concerned with the portfolio selection; upper, lower bounds and a linear constraint that requires sum of countries equal 1. Then, based on the proposed framework of the ICA and country representation of portfolio, computational analyses were provided so as to investigate the performance of the ICA-based approach for the portfolio selection problem and also to evaluate the validity of this approach. Thus, a portfolio was selected from Tehran Stock Exchange market, from 50 top companies. To investigate its performance and efficiency, it was applied to the return data of companies for a test-period of six months, which followed the portfolio construction historical data. The results indicate that the ICA-based portfolio has a better performance in contrast to its market counterpart, *i.e.* in contrast to the market portfolio. ICA portfolio outperformed the market portfolio's average return, 67% of the times during the six month test period. The proposed ICA-based approach is considered efficient since it never misses the optimal solution even when it was run repeatedly (twenty times). Moreover, the time for every iteration remains the same. In addition, ICA portfolio approach was able to locate the solution in complicated and large search space of the real stock market efficiently; the convergence speed for locating the solution, present the method suitable and with fast performance. On the whole, the results indicate that the proposed approach can consistently handle the practical portfolio selection problem with good efficiency.

More work is currently underway to develop a bi-criterion ICA algorithm to construct the entire Pareto front for the risk-return tradeoff analysis. Another experiment is to enhance the efficiency of the current ICA algorithm. Consideration of different time steps for reporting rates of returns -periods like every 3 or 6 months- to construct the covariance and hence the portfolio, could be another subject for further investigations.

REFERENCES

1. Hagin, R.L., 1979. The Dow Jones-Irwin Guide to Modern Portfolio Theory. Homewood, Illinois: Dow Jones-Irwin.
2. Brealey, R.A., 1969. An Introduction to Risk and Return from Common Stock Prices. M.I.T Press.
3. Evans, J.L. and Stephen H. Archer, 1968. Diversification and the Reduction of Dispersion: An Empirical Analysis. J. Finance, 23(12): 761-767.
4. Gaumnitz, J.E., 1967. Investment Diversification under uncertainty: An Examination of the Number of Securities in a Diversified Portfolio. Stanford University.
5. Markowitz, H., 1952. Portfolio Selection. J. Finance, 7(1): 77-91.
6. Hagin, R.L., 2004. Investment Management - Portfolio Diversification, Risk and Timing - Fact and Fiction, ed. W. finance. Hoboken, New Jersey: John Wiley and Sons, Inc.
7. Talebi, A. and M.A. Molaei. 2010. Application of Genetic Algorithm in Portfolio Optimization. in The 1st International Conference of Business and Economics. Thessaloniki, Greece.
8. Holland, J., 1975. Adaptation in natural and artificial systems. Massachusetts: MIT press.
9. Chaiyaratana, N. and A.M.S. Zalzala, 1997. Recent developments in evolutionary and genetic algorithms: theory and applications. in The Second international conference on genetic algorithms in engineering systems: Innovations and Applications.
10. Bi, J. and A. Eladdadi, 2001. Application of Optimization Algorithms in Portfolio Selection Problems, in Department of Mathematical Sciences Rensselaer Polytechnic Institute: Troy, New York.
11. Chan, M., C. Wong, B.K.S. Cheung and G.Y.N. Tang, 1999. Genetic Algorithms in Multistage Portfolio optimization System. Society for Computational Economics, pp: 165.
12. Fichter, D.P., 2000. Application of Genetic Algorithm in Portfolio Optimization for the Oil and Gas Industry, in SPE Annual Technical Conference and Exhibition. 2000, Society of Petroleum Engineers Inc.: Dallas, TX.
13. Lazo, J.G., M. Maria, R. Vellasco, M. Auelio and C. Pacheco. 2000. A Hybrid- Genetic-Neural System for Portfolio Selection and Management. in Sixth Int. Conf. On Engineering Applications of Neural Networks, EANN. Kingston upon Thames.
14. Loraschi, A., A. Tettamanzi, M. Tomassini, P. Verda, D.W. Pearson and R.T. Albercht. 1995. Distributed Genetic Algorithm with an Application to Portfolio Selection Problem. in Int. Conf. On Artificial Neural Nets and Genetic Algorithms. 1995. Springer-Verlag.
15. Schlottmann, F. and D. Seese, 2001. A. Hybrid Genetic Quantitative Method for Risk-Return Optimization of Credit Portfolio, in Faculty of Economics. University of Karlsruhe, Institute AIFB: Karlsruhe, Germany.
16. Shapcott, J., 1992. Index Tracking: Genetic Algorithm for Investment Portfolio Selection, E.P.C. Center, Editor. 1992, University of Edinburgh: Edinburgh.
17. Vieira, R.G.S. and R.S. Wazlawick, Using Genetic Algorithm for Defining an Initial Shares Portfolio, in World Multi Conference on Systemic, Cybernetics and Informatics(CSI'98) and the 4th International Conference on Information Systems Analysis and Synthesis (ISAS'98). 1998-1999, INE, Federal University of Santa Catarina: Orlando, U.S.A. pp: 333-338.
18. April, J., F. Glover and J. Kelly, 2002. Portfolio Optimization for Capital Investment Projects. in Winter Simulation Conference.
19. Demiguel, A.V. and R. Uppal, 2004. Portfolio Investment with the Exact Tax Basis via Non-Linear Programming. J. Management Sci., 51(2): 277-290.

20. Gondzio, J. and A. Grothey, 2004. Solving Nonlinear Portfolio Optimization Problems with the Primal-Dual Interior Point Method.
21. Hershberger, M., Y. Qi and R.E. Steuer, 2004. Quadratic Parametric Programming for Portfolio Selection with Random Problem Generation and Computational Experience.
22. Inuiguchi, M. and J. Ramik, 2000. Possibilistic Linear Programming: A Brief Review of Fuzzy Mathematical Programming and A Comparison with Stochastic Programming in Portfolio Selection Problem. Fuzzy Sets and Systems J., 111: 3-28.
23. De Klerk, E. and D.V. Pasechnik, 2005. A Linear Programming Reformulation of the Standard Quadratic Optimization Problem.
24. Li, Z.F., S.Y. Wang and X.T. Deng, 2000. A Linear Programming Algorithm for Optimal Portfolio Selection with Transaction Cost. International J. Systems Sci., 31: 107-117.
25. Lobo, M.S., M. Fazel and S. Boyd, 2002. Portfolio Optimization with Linear and Fixed Transaction Costs.
26. Loraschi, A. and A. Tettamanzi, 1995. An Evolutionary Algorithm for Portfolio Selection in a Downside Risk Framework. European J. Finance, 1: 1.
27. Papahristodoulou, C. and E. Dotzauers, 2004. Optimal Portfolios Using Linear Programming Models. J. the Operational Research Society,
28. Parra, M.A., A.B. Terol and M.V. Rodriguez, *A Fuzzy Goal Programming Approach to Portfolio Selection.* European Journal of Operational Research, 2001. 133(2): p. 287-297.
29. Ramaswamy, S., 1998. Portfolio Selection Using Fuzzy Decision Theory.
30. Schubert, L., 2005. Performance of Linear Portfolio Optimization.
31. Sen, S., L. Yu and T. Genc., 2006. A Stochastic Programming Approach to Power Portfolio Optimization. Operation Research J., 54(1): 55-72.
32. Van Le, T., Fuzzy Evolutionary Programming for Portfolio Selection in Investment, in Second Int. Conf. In Intelligent Processing and Manufacturing of Materials. 1999: Honolulu. pp: 675-679.
33. Yang, X., 2006. Improving Portfolio Efficiency: A General Algorithm Approach. J. Computational Economics, 28(1): 1-14.
34. Zenios, S.A., M. Holmer, R. McKendall and C.H. Vassiadou-Zeniou, 1998. Dynamic Models for Fixed-Income Portfolio Management under Uncertainty. Journal of Economic Dynamics and Control, 22(10): 1517-1541.
35. Fogel, D.B., 1995. Evolutionary computation toward a new philosophy of machine intelligence. New York: IEEE Press.
36. Eberhart, R.C. and Y. Shi, 1998. Comparison between genetic algorithms and particle swarm optimization. in seventh annual conference on evolutionary programming, Evolutionary programming VII. San Diego, CA.
37. Tushar, J. and M.J. Nigam, 2010. Synergy of evolutionary algorithm and socio-political process for global optimization. Expert Systems with Applications, 37: 3706-3713.
38. Atashpaz-Gargari, E. and C. Lucas, 2007. Imperialist Competitive Algorithm: An algorithm for optimization inspired by imperialistic competition. in IEEE Congress on Evolutionary Computation.
39. Kirkpatrick, S., C.D. Gelatt Jr. and M.P. Vecchi, 1983. Optimization by simulated annealing. Sci., 220: 671-680.
40. Eberhart, R.C. and J. Kennedy. 1995. A new optimizer using particle swarm theory. in The Sixth International Symposium on Micromachine and Human Science. Nagoya, Japan.
41. Kennedy, J. and R.C. Eberhart, 1995. Particle swarm optimization. in The IEEE International Joint Conference on Neural Networks. Piscataway, NJ.: IEEE Press.
42. Dorigo, M. and V. Maniezzo, 1996. Colorni A. The ant system: optimization by a colony of cooperating agents. IEEE Trans. Syst. Man Cybernet. B, 26(1): 29-41.
43. Rajabioun, R., F. Hashemzadeh, E. Atashpaz-Gargari, B. Mesgari and F. Rajaei Salmasi, 2008. Identification of a MIMO Evaporator and Its Decentralized PID Controller Tuning Using Colonial Competitive Algorithm, in IFAC World Congress.
44. Lucas, C., Z. Nasiri-Gheidari and F. Tootoonchian, Application of an imperialist competitive algorithm to the design of a linear induction motor. Energy Conversion and Manage., 51: 1407-1411.
45. Sayadnavard, M., 2010. Localization in Wireless Sensor Networks based on Imperialist Competitive Algorithm, in The 13th Conference on Network-based Information Systems.
46. Karimi, S., 2009. Shahriar Baradaran Shokouhi, Mehdi Mokhtarzade, Optimal Hierarchical Remote Sensing Image Clustering Using Imperialist Competitive Algorithm, in 2nd International Conference on Machine Vision (ICMV).

47. Bagheri, A., G. Ghodrati Amiri and M. Khorasani, 2009. Prediction of horizontal peak ground and spectral accelerations using Genetic and Imerialist Competitive Algorithms. J. Earthquake Engineering,
48. Kaveh, A. and S. Talatahari, 2010. Optimum design of skeletal structures using imperialist competitive algorithm. Computers and Structures, 88(21-22): 1220-1229.
49. Kaveh, A. and S. Talatahari, 2010. Imperialist Competitive Algorithm for Engineering Design Problems. Asian J. Civil Engineering (Building and Housing), 11(6): 675-697.
50. Biabangard-Oskouyi, A., E. Atashpaz-Gargari, N. Soltani and C. Lucas, 2009. Application of imperialist competitive algorithm for material properties characterization from sharp indentation test. Int J Eng Simul, 10: 1.
51. Tayefeh, M., 2009. Nafiseh Mahmoodi, Fattaneh Forouzideh, Caro Lucas, Taghiyareh, Artificial Neural Network Weights Optimization based on Imperialist Competitive Algorithm, in Computer Science and Information Technologies (CSIT 2009) and 4th International Scientific and Technical Conference. Lviv, Ukraine.
52. Zarandi, F.M., A. Molladavoudi and A. Hemmati, 2010. Fuzzy time series based on defining interval length with Imperialist Competitive Algorithm. in Fuzzy Information Processing Society (NAFIPS), Annual Meeting of the North America. 2010: IEEE.
53. Jasour, M.A., E. Atashpaz-Gargari and C. Lucas, Vehicle Fuzzy Controller Design Using Imperialist Competitive Algorithm. in 2nd Iranian Joint Conference on Intelligent and Fuzzy Sysytems. 2007. Mashhad, Iran.
54. Rajabioun, R., E. Atashpaz-Gargari and C. Lucas, Colonial Competitive Algorithm as a Tool for Nash Equilibrium Point Achievement. Lecture notes in computer Sci., 5073: 680-395.
55. Hosseini, I., Mohsen Nasab, Morteza Khezri, Sahab Khodamoradi and E. Atashpaz-Gargari, An Application of Imperialist Competitive Algorithm to Simulation of Energy Demand based on Economic Indicators: Evidence from Iran. Europian J. Scientific Res., 43(4): 495-506.
56. Sepehri Rad, H. and C. Lucas, 2008. Application of Imperialistic Competition Algorithm in Recommender Systems, in 13th Int'l CSI Computer Conference (CSICC'08). Kish Island, Iran.
57. Tamimi, A., Houman Sajadian and Hesam Omranpour, 2010. Mobile Robot Global Localization Using Imperialist Competitive Algorithm, in The 3rd International conference on Advanced Computer Theory and Engineering.
58. Jolai, F., M.S. Sangari and M. Babaie, 2009. Pareto simulated annealing and colonial competitive algorithm to solve an offline scheduling problem with rejection. Engineering Manufacture, pp: 224.
59. Shokrollahpour, E., M. Zandieh and B. Dorri, 2010. A novel imperialist competitive algorithm for bi-criteria scheduling of the assembly flowshop problem. International J. Production Res., pp: 1-17.
60. Nazari-Shirkouhi, S., H. Eivazy, R. Ghodsi, K. Rezaie, E. Atashpaz-Gargari, 2010. Solving the integrated product mix-outsourcing problem using the Imperialist Competitive Algorithm. Expert Systems with Applications,
61. Forouharfard, S. and M. Zandieh, 2010. An imperialist competitive algorithm to schedule of receiving and shipping trucks in cross-docking systems. The International J. Advanced Manufacturing Technol., pp: 1-15.
62. Bagher, M., M. Zandieh and H. Farsijani, 2010. Balancing of Stochastic U-type Assembly lines: An Imperialist Competitive Algorithm. International J. Advanced Manufacturing Technol.,
63. Sarayloo, F. and R. Tavakkoli-Moghaddam, 2010. Imperialistic Competitive Algorithm for Solving a Dynamic Cell Formation Problem with Production Planning. Advanced Intelligent Computing Theories and Applications, 6215: 266-276.
64. Taheri, A.R., 2009. Human Socio-political Evolution and Imperialistic Competitive Algorithm Application in Optimize the Huge-Combinatorial problem, in The 2nd International Conference of Iranian Operations Research Society. Babolsar, Iran.
65. Markowitz, M.H., 1959. Portfolio Selection: Efficient Diversification of Investments. New York: John Wiley and Sons.
66. Sharpe, W.F., Investments. 1978, Englewood Cliffs, U.S.A: Prentice-Hall, Inc.
67. Williams, J.B., The Theory of Investment Value. 1938, Cambridge, Mass.: Harvard University Press.
68. Parker Jones, C., Investments: Analysis and Management. 1999: Wiley finance.
69. Haupt, R.L. and S.E. Haupt, Practical Genetic Algorithms. 2nd ed. ed. 2004, Hoboken, NJ.: Wiley.

70. Bonabeau, E., M. Dorigo and G. Th´eraulaz, From Natural to Artificial Swarm Intelligence. 1999, New York: Oxford University Press.
71. Kennedy, J. and R.C. Eberhart, Swarm Intelligence. 2001: Morgan Kaufman.
72. Bishop, C.M., Neural Networks for Pattern Recognition. 1995: Oxford University Press.
73. Fiesler, E. and R. Beale, Handbook of Neural Computation. 1996: IOP Publishers and Oxford University Press.
74. Back, T., Evolutionary Algorithms in Theory and Practice, Evolution Strategies, Evolutionary Programming, Genetic Algorithms. 1996, New York, Oxford: Oxford University Press.
75. Back, T., David B. Fogel and Zbigniew Michalewicz, Handbook of Evolutionary Computation. 1997: IOP Publishers and Oxford University Press.
76. Back, T., David B. Fogel and Zbigniew Michalewicz, Evolutionary Computation 1 Basic Algorithms and Operators. 2000, INSTITUTE OF PHYSICS PUBLISHING: Bristol and Philadelphia.
77. Dorigo, M., T. Stu¨ tzle, Ant Colony Optimization. 2004: MIT Press.
78. Engelbrecht, A.P., Fundamentals of Computational Swarm Intelligence. 2005: John Wiley and Sons.
79. The MathWorks, I., Matrix laboratory (MATLAB). 1984-2009 ©The MathWorks, Inc.
80. Atashpaz-Gargari, E., F. Hashemzadeh, R. Rajabioun and C. Lucas, 2008. Colonial Competitive Algorithm, a novel approach for PID controller design in MIMO distillation column process. International Journal of Intelligent Computing and Cybernetics, 1(3): 337-355.
81. Solnik, B.H., 1974. The International Pricing of Risk: An Empirical Investigation of the World Capital Market Structure. J. Finance, 29(2): 364-378.
82. Solnik, B.H., 1974. Why Not Diversify Internationally Rather Than Domestically? Financial Analysis J., 30(4): 45-54.