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# Optimal Distributed Generation Location in Radial Distribution Systems Using A New Heuristic Method

[1]H. Chahkandi Nejad, [2]J. Olamaei, [3]R. Jahani, [3]H. Ghiasi Zadeh

[1]Department of Electrical Engineering, Islamic Azad University, Birjand Branch, Birjand, Iran.
[2]Islamic Azad University, South Tehran Branch, Tehran, Iran.
[3]Islamic Azad University, Birjand Branch, Birjand, Iran.

**Abstract:** Heuristic Search and PSO optimization methods and Imperialist Competitive Algorithm(ICA) to optimally determine distributed generation location and size are compared in a distribution network. Objective function consists of power losses and improvement in voltage profile and applied methods are tested on the IEEE 33-bus system. This study demonstrates comparson results of proposed approaches. It includes determination of required DG being installed in an appropriate location. This study also demonstrates that system losses may increase if DG units are connected at non-optimal locations or have non-optimal size.Results indicate that by ICA method, better results are obtained in comparison to PSO and the simple heuristic search method on the 33-bus radial distribution systems. By ICA, maximum loss reduction for optimally placed multi-DGs is determined. furthermore, voltage profile improvement and branch current reduction are achieved.

**Key words:** Distributed Generation; Optimal DG size; Imperialist Competitive Algorithm(ICA); Particle Swarm Optimization (PSO), Heuristic Search (HS).

## INTRODUCTION

The term distributed power generation (DG) is to install small generating units near load centers, in order to prevent network expansion in and to cover new load areas or increased energy to satisfy consumers demand. To simplify that, Distributed Generation (DG) is a small generator spotted throughout a power system network, providing local electricity to load customers (Thomas Ackermann., *et al*. 2001). DG's are alternative for industrial, commercial and residential applications. DG's rely on technology which is efficient, reliable, and simple so that it can competes with traditional large generators in some areas (W.El-Khattam & M.M.A.Salama, 2004). Artificial intelligence techniques are most widely applied tools in most optimization problems. These methods (e.g. genetic algorithm, simulated annealing and tabu search) seem to be promising and are still running.DG allocation by application of genetic algorithm (GA) (G. Celli & F. Pillo, 2001) is studied. Tabu search (TS) algorithm is used for distribution systems DG allocation (K. Nara,., *et al*. 2001). Analytical approaches minimizing line losses are also applied in DG allocation as provided in (C. Wang & M. H. Nehrir, 2004). In (A. Keane & M. O' Malley, 2005),authors have integrated DG in distribution systems using power systems studies coupled by means of linear programming method. Papers (L. F. Ochoa., *et al*. 2005) utilize evolutionary programming for placement identification of DG in distribution systems.we have considered DG's real power supply while consuming proportionately reactive power.Newton-Raphson algorithm based load flow program is used to solve the load flow problem In (Naresh Acharya). Optimal placement methodology for DG is proposed. Furthermore, the heuristic search requires exhaustive search for all possible locations which has probably no application to more than one DG. Therefore, Imperialist Competitive Algorithm (ICA) is fully practiced to determine multi-DGs'optimal location and sizes for total real power loss and investment costs minimization and to improve voltage profile of the distribution systems. A backward-forward sweep based load flow program (D. Shirmohammadi & C.S. Cheng, 1995) is applied for the load flow problem solving. The following shows different parts of the proposed paper: Section 2 addresses formulation of the problem. DG Information is explained in Section 3. Section 4 represents The ICA. Simulation result on the test systems are illustrated in Section 5, and finally in Section 6 author's conclusion is presented.

***Problem Formulation:***

three main objectives of the minimized function are:

**Corresponding Author:** J. Olamaei, Islamic Azad University, South Tehran Branch, Tehran, Iran.

***II.1. Minimize the Active Power Losses:***

Real power loss reduction in a distribution system is required efficient power system operation.System loss can be calculated frmo equation (1), giving system operating condition,

$$P_L = \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij}(P_iP_j + Q_iQ_j) + B_{ij}(Q_iP_j - P_iQ_j) \quad (1)$$

Where

$$A_{ij} = \frac{R_{ij}\cos(\delta_i - \delta_j)}{V_iV_j}$$

$$B_{ij} = \frac{R_{ij}\sin(\delta_i - \delta_j)}{V_iV_j}$$

Where, $P_i$ and $Q_i$ are net real and reactive power injected in bus *'i'*respectively, $R_{ij}$ is the line resistance between bus *'i'* and *'J'*, $V_i$ and $\delta_i$ are the voltage and angle at bus *'i'* respectively. Placement technique objective is to minimize total real power loss. Mathematically, the objective function can be written as:

$$P_L = \sum_{k=1}^{N_{sc}} Loss_k \quad (2)$$

Subject to power balance constraints

$$\sum_{i=1}^{N} P_{DGi} = \sum_{i=1}^{N} P_{Di} + P_L \quad (3)$$

Where: $Loss_k$ is distribution loss at section *k*, $N_{SC}$ is total number of sections, $P_L$ is the real power loss in the system, $P_{DGI}$ is the real power generation DG at bus *i*, $P_{Di}$ is the power demand at bus *i*.

***II.2. Voltage Profile Improvement:***

$$F_2 = \sum_{i=1}^{N} | V_i - V_{i,ref} | \quad (4)$$

Voltage constraints:

$$| V_i |^{\min} \le | V_i | \le | V_i |^{\max} \quad (5)$$

Current limits:

$$| I_{ij} | \le | I_{ij} |^{\max} \quad (6)$$

***DG Type:***

It's obvious that a DG will supply real power and in turn and absorbs reactive power. In case of wind turbines, induction generator is used to produce real power and the reactive power to be consumed in the process (D. Shirmohammadi & C.S . Cheng, 1995). The amount of reactive power they require is an ever increasing function of the active power output. Reactive power consumed by the DG (wind generation) in given as in equation (7) as stated in (R.H. Lasseter., 1998).

$$Q_{DG} = -(0.5 + P_{DG}^2) \quad (7)$$

By loss equation modification. And following similar methodology of the first two types, optimal DG size is determined by solving equation (8).

$$P = 0.0032 A_{ii} P_{DGi}^3 + P_{DGi}[1.004 A_{ii} + 0.08 A_{ii} Q_{Di} - 0.08 Y_i] + (X_i - A_{ii} P_{Di}) = 0 \quad (8)$$

Equation (8) shows the real power that a DG is to produce when located at but *'i'*, to obtain the minimum system loss while the amount of consumed reactive power can be can be determined from equation (7).

***ICA algorithm introduction:***

Fig 1 shows ICA algorithm flowchart. This algorithm, such as other evolutionary algorithms begins with some accidental primary crowds that each of them has been called a "country". Some of best elements of crowds are selected as imperialist (equal with elites in genetic algorithms). the remaining crowds have been considered as colony. Imperialist, with their power, absorb these colonies to themselves with special trend that will be discussed at future. Power of each empire depends on its two constitutive part namely imperialist country (as central nucleus) and its colonies. In mathematics this dependence models with empire power definition in the form of power sum of imperialist country plus percents of average power of its colonies. The imperialist competition between them begins with forming early empires. Each empire that cannot be successful in imperialist competition and increases its power, will be removed from imperialist competition scene .therefore the survival of each empire depends on its power in absorption of revival empire's colonies and ruling over them. As a result, in imperialist competition streams, the power of greater empires will be increased and weak empires will be removed. Empire will be obliged improve their colonies for increasing their power. Colonies gradually near the empires and we can observe some sort of convergence. Final extent of imperial extent is when we have had unit empire in the world, with colonies which are close to the imperialist country accordance with their position.For starting the algorithm, we create N numbers of early countries .we select N imp of the best members of this crowd as imperialist (the countries including minimum amount of cost function), the remains forms N col of colonies countries in which each of them belongs to one empire. We give some of these colonies to each imperialist for dividing the early colonies among the imperialist accordance with their power. consider their normalized cost as follow:

$$C_n = \max\{c_i\} - c_n \tag{9}$$

Where $c_n$ imperialist cost max ($c_i$) is highest cost among imperialist and $C_n$ is normalized cost of this imperialist.

Each imperialist which have had more cost (be weaker imperialist), includes less normalized costs. Normalized respective power of each imperialist, with having normalized costs, has been calculated as follow and accordance with it, colonies countries have been divided between imperialist.

$$P_n = \frac{C_n}{\sum_{i=1}^{N_{imp}} C_i} \tag{10}$$

From other respect, normalized power of an imperialist is colonies proportion that are controlled by that imperialist.Therefore the early number of an imperialist's colonies equals with:

$$N.C.n = \text{round}\{P_n.(N_{col})\} \tag{11}$$

Where NC.n is early number of empire 's colonies and $N_{col}$ is the total number of existing colonies countries in the early countries crowds. Round is also function that give closest integer t a decimal number. We select accidentally some of these primary colonies countries, with considering N.C for each empire an give it to N imperialist, the imperialist competitive algorithm begins with having primary status of all empires. Evolutionary trend which located in a segment that continues till the stop condition fulfillment. Fig 2 shows the manner of early empires forming. Bigger empires have more colonies. In this Fig, imperialist number 1 creates the strongest empire and have most number of colonies.

***Iv.1. Absorption Policy Modeling:***

Colonies movement toward the assimilation policy of imperialist has done with the purpose of analyzing the culture and social structure of colonies in central government culture. Imperialist countries began to creating development (building transportation substructure, university establishing ,…).

In fact this central government tries to close colony country to its self by applying attraction policy, in different political and social dimensions, with considering showing manner of country in solving optimization problem. This section of imperialistic process in optimization algorithm has been modeled in the form of colonies movement toward the imperialist country. The Fig 3 shows total image of this movement. According with this Fig, imperialist country attract to itself parallel with culture and language axis. As shown in this Fig, colony country moves in x unit size toward the attachment line of colony to the imperialist and drawn to new situation. In this Fig, distance between imperialist and colony is shown by D, and x is accidental number with steady distribution.

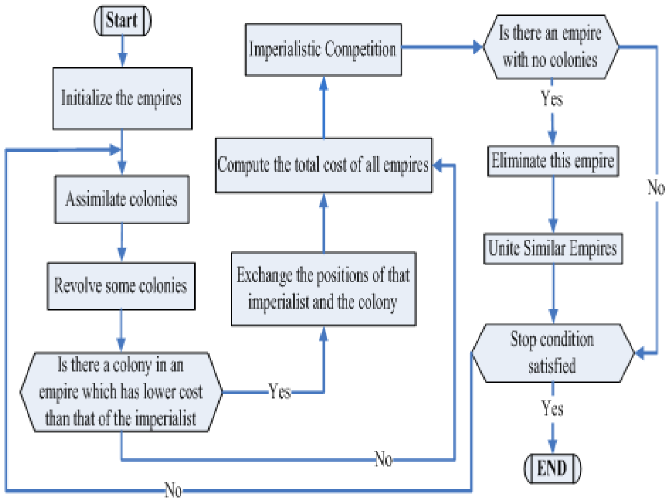


**Fig. 1:** Flowchart of the Imperialist Competitive Algorithm

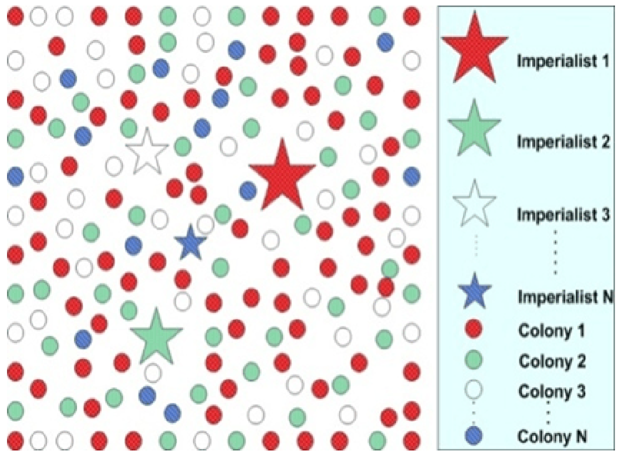


**Fig. 2:** Manner of forming primary empire, imperialist number 1 creates strongest empire and has maximum number of colonies.

It means for x, we have

$$x \sim U(0, \beta \times d) \tag{12}$$

Where β is a number bigger than 1 and nears to 2. A good selection can be β=2. The existence of coefficient β□1 causes the colony country closes to the imperialist country from different aspects while moving. With historical survey of assimilation phenomena, one clear fact in this field is in spite that imperialist countries followed seriously the attraction policy but facts did not follow totally accordance with applied policy and there were deviances in the work results. In introduced algorithm, this probable deviation has done with adding an accidental angle to the attraction path of colonies. For this purpose, in the colonies movement toward the imperialist, we add an accidental angle toward the colony movement, Fig 4 shows this state. this time we continue our path in stead of x movement toward the imperialist and in toward the vector and colony maxim to the imperialist in the same extent, but with θ deviation in the path, and consider θ accidentally and with constant distribution (but any ideal and proper distribution can be used), then $\theta \sim U(-\gamma, \gamma)$

In this relation y is ideal parameter that its increasing causes increasing searching around imperialist and its decreasing causes colonies close possibly to the vector of connecting colony to the imperialist. With considering the radian unit for θ, a number close to π/4 was proper selection in the most depletion.

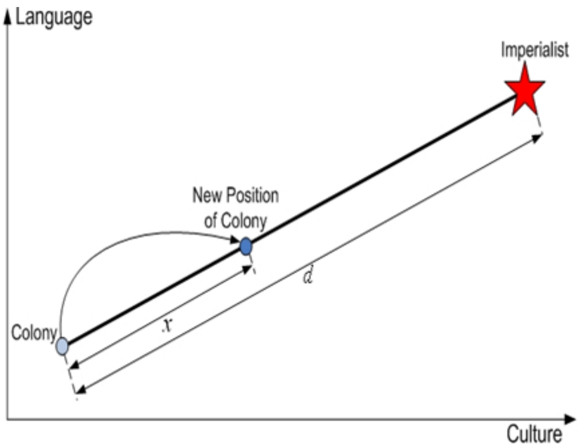


**Fig. 3:** total image of colony movement toward imperialist.

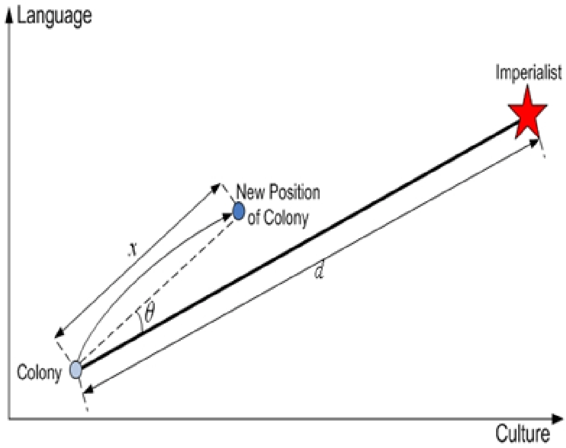


**Fig. 4:** real movement of colonies toward the imperialist.

***Position Displacement of Colony and Imperialist:***

In some cases attraction policy has had possitive result for them, in spite of destroying political-economical structures of colony countries. some of countries with appling this policy accessed to general self confidence and after awhile it was the educated people who combat with the nation leadership for escaping from imperialist. We can find various cases of these in enland and france' s colonies. from other perspective, looking at up and downs of power circulation in the countries shows truly that the countries in which were at the peak of politic military power, after awhile declined and contrary the countries reached to the power that before were not into the power. This historical movement in the modelling in the algorithm has been applied in the way of colony movement toward the imperialist country, some of these colonies may reach to a better condition than imperialist (reaching to the points in cost function that generate less costs than cost function extent ). In this state, the imperialist country and colony change their position and algorithm continues with imperialist counry in new situation and this time it is the new imperialist country in which begin to applying assimilation policy for its colonies. The colony and imperialist displacement is shown in the Fig 5. In this Fig the best empire's colony in which has less costs than imperialist, is shown with dark colour. Fig 6 shows the whole empire after position changing.

***Total Power of an Empire:***

The power of an empire equals with the power of imperialist country in addtion to some perceptage of total power of whole colonies, in this case the total cost of an empire calculate as follow:

$$T.C. = Cost(imprialist_n) + \xi mean\{Cost(colonies\ of\ empire_n)\} \tag{13}$$

Where T.C.n is the empire's total cost and ζ is possitive number that is usually between zero and one and near to zero.This low considering of ζ cause total cost of empire be nearry equal with its central government and inreasing ζ causes increasing the colonies's costs measure infulunce of an empire in determining the its total costs. In generic state ζ =0/ 05 in the most cases reulted to proper answers.

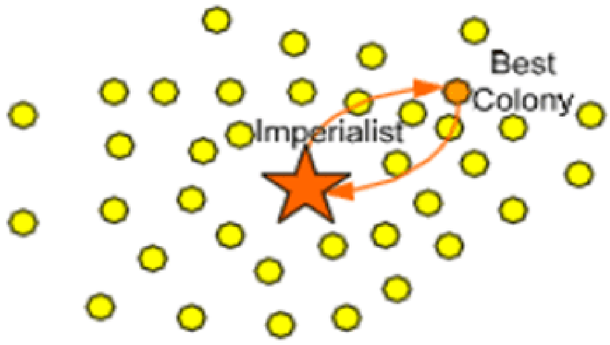


**Fig. 5:** displacement of colony and imperialist

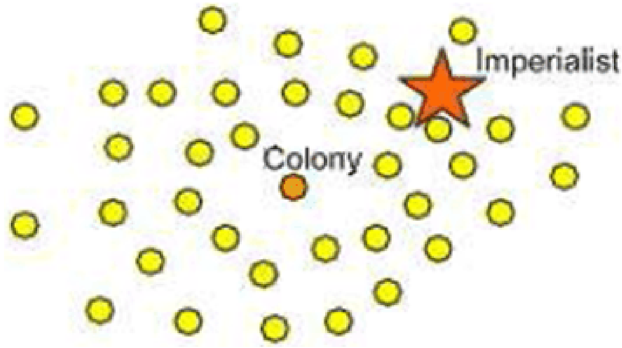


**Fig. 6:** the whole empire after displacement

***Imperial competition:***

Each empire which cannot increase its power and looses its competiton power, will be removed from imperialistic competitions. this removing forms gradually. it means that with passing the time, weak empire give up their colonies and the strong empire take possession of these colonies and increase their power. for modelling this fact, we assume the empire at the time of deleting, is the weakest existing empire. So in the algghorithm repetition, we take some of weakest colonies of the empire and create a competition between the whole empires. mentioned colonies will not nessesarily be possessed the strongest empire. But this is the stonger empire which has more chance for its ownership. Fig 9 shows total image of this part of alghorithm

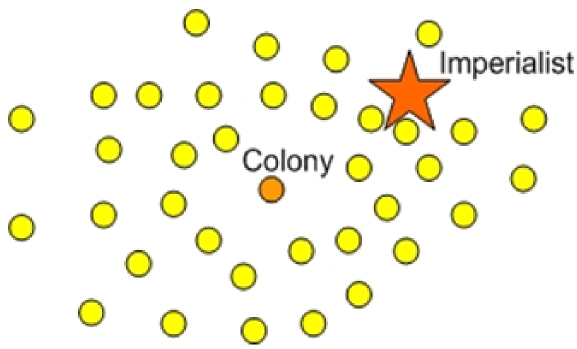


**Fig. 7:** Exchanging the positions of a colony and the imperialist

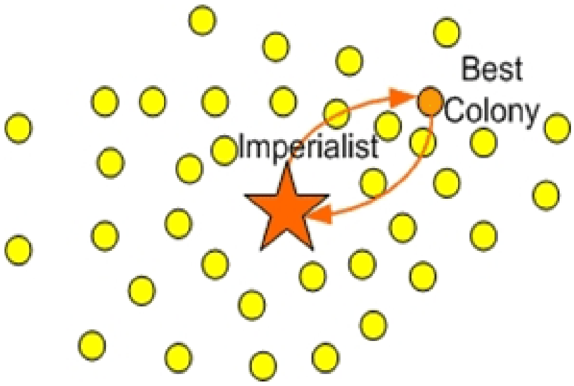


**Fig. 8:** The entire empire after position exchange

For modelling the competition between the empires for possessing these colonies, first of all we calculate the owenership probability of each empire(that be fit with the power of that empire) with considering total cost of each empire, as follow: first we determine total costs of empire based on its nnormalized costs:

$$N.T.C._n = T.C._n - \max_i \{T.C._i\} \quad (14)$$

Where T.C.N is n total cost of empire and N.T.C.n is normalized costof that empire. Each emire which have had less T.C.N, has more n.t.cn. infact T.C .n equals total cost of an empire and N.T.C. N equalsits total power.

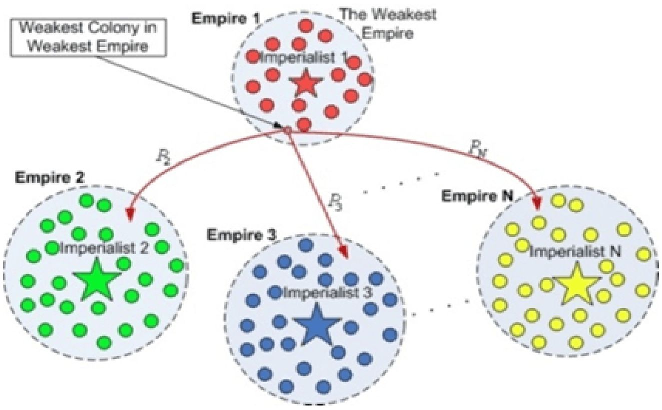


**Fig. 9:** total image of imperialistic competition: bigger empire take the possession oof the other emire's colonies with more liklihood.

The probability of colony ownership in competition by each empire calculates as follow:

$$p_{p_n} = \left| \frac{N.T.C._n}{\sum_{i=1}^{N_{imp}} N.T.C._i} \right| \quad (15)$$

with ownership probability of each empire, we divide the mentioned colonies accidentally between the empires, but with related probability to ownership probability of each empire. Then we form vector P based on above probabiliy extents as follow:

$$\mathbf{P} = \left[ p_{p_1}, p_{p_2}, p_{p_3}, ..., p_{p_{N_{imp}}} \right] \quad (16)$$

vector P' s size is 1×nimp and is consituted based on probability amounts of empires ownership. Then we form the accidental vector R as equal as vector P, the arrays of this vector are accidental number with the same distribution in [0,1].

$$R = \left[ r_1, r_2, r_3, \ldots, r_{N_{imp}} \right]$$
$$r_1, r_2, r_3, \ldots, r_{N_{imp}} \sim U(0,1) \tag{17}$$

Then we form vector D as follow

$$\mathbf{D} = \mathbf{P} - \mathbf{R} = \left[ D_1, D_2, D_3, \ldots, D_{N_{imp}} \right]$$
$$= \left[ p_{p_1} - r_1,\ p_{p_2} - r_2,\ p_{p_3} - r_3\ , \ldots\ , p_{p_{N_{imp}}} - r_{N_{imp}} \right] \tag{18}$$

We give the mentioned colonies to the empires with having cector D so that related andis in vector D be bigger than others. The empire which has more ownership probability, has the highest extent ,with more chances in related andis in vector D.

***Declining the weak Impires:***

Weak empires gradully decline in imperialistic competition and strong empires take the possession of their colonies. There are different conditions for declining an empire. In suggested empire, when an empire loose its colonies, it assumped deleted.

***Convergency:***

The mentioned alghorithm continues till fulfillment of one convergency condition or afinishing the number of whole repitition. After awhile all the empires will decline and we have only one empire and other countries are under the control of this united empire. In this new ideal world all the colonies are controled bye an united empire and the colonies 's cost and situations equals with the empirilaist 's cost and situation. in this new world, there are no difference not only between colonies but also between colonies and imperialist country. in other words, all the countries are both colony and imperialist at the same time. In such situation the imperialistic competition have been finished and stops as one stop codition of algorithm.

***ICA alghorithm:***

Assimilation: this function applies assimilation part or in other word attraction policy. Primary empires: it forms primary empires with proper dividing of colonies among them, with concidering situation and cost of primary countries. Imperialistic competition: The imperialistic competition between the emipres in order to attract each other colonies is done by this function. removing the weak empires is also in this function. Imperialist and colony displacement: Displacement of imperialist and colony is done in this function. If a colony reach to a better position than imperialistt ,it immediately take the control of emperor and continues the work with applying the attraction policy on them.The colonies revolution: Revelution, that is main counterweight of discovery balance and exploitation and is useful for discovery, applies in this function. sudden changes happens in some countries and in some cases leads to discovery of minimum indiscernible point in function.

***ICA Alghorithm's Similar_code:***

1. Select some accidental point on the function and form the primirary empires. we mean the powerhouses power that are concidered as primary guess.
2. Move the colonies toward the imperialist country (assimilation policy ).
3. If there are an empire that has less costs than imperialist ,change the position of colony and imperialist.
4. Calculate total costs of an empire(with pay attention to imperialist and its colonies's costs).
5. Select one colony from weakest empires an give it to the empire which has more chance for ownership.
6. Delete weak empires.
7. Stop if there are only one remained empire, otherwise go to 2.

***Simulation Results:***

The distribution test systems are the 33 bus systems (M. Ermis, *et al*. 1992). The original total real power loss and reactive power loss in the system are 221.4346 kW and 150.1784 kVar, respectively. The 33 bus system has 32 Sections with the total load of 3.80 MW and 2.33 MVar, shown in Figure 10. The original total real and reactive power losses of the system are 230.0372 kW and 104.3791 kVar, respectively.

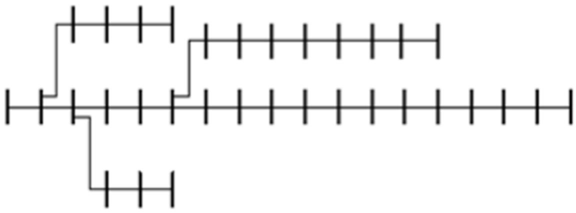

**Fig. 10:** The 33 bus radial distribution system.

For DG, The average CPU time is 4.8502 second.

**Table 1:** PSO result of the 33 bus test system

| Algorithms | Total Power Loss (kW)(Min) | Total Power Loss (kW) (Avg) | Total Power Loss (kW) (MAX) | Average Time (sec) |
|---|---|---|---|---|
| heuristic search | 164.6975 | 177.4055 | 286.8644 | 5.6135 |
| PSO | 162.1489 | 175.8621 | 203.7605 | 5.1611 |
| ICA | 158.7388 | 171.8713 | 277.1935 | 4.8502 |

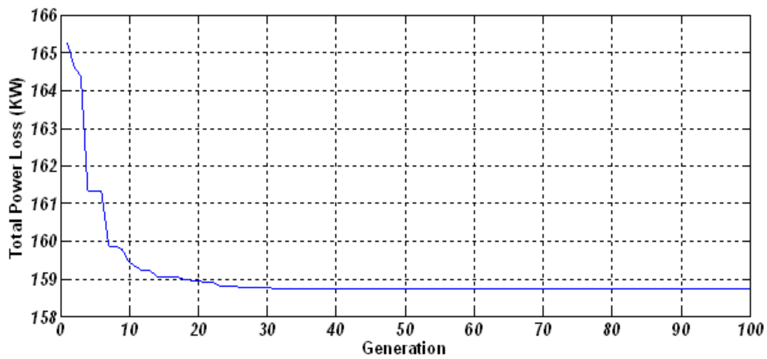


**Fig. 11:** Convergence characteristic of the 33 bus test system.

**Table 2:** Optimal DG placement for DG

| system | Method | Bus No. | DG Size MW | Bus No. | DG Size MW | Bus No. | DG Size (MW) | $P_{LOSS}$ (Kw) | $Q_{LOSS}$ (Kw) | Loss Reduction% Real | Reactive |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Load Flow Analysis | - | - | - | - | - | - | 221.4342 | 150.1783 | - | - |
| | Heuristic Search | 14 | 1.9933 | - | - | - | - | 163.8515 | 115.5283 | 27.03 | 23.08 |
| | PSO | 14 | 1.9354 | - | - | - | - | 163.8976 | 115.9445 | 26.03 | 23.08 |
| 33 bus | PSO | 3 | 1.3163 | 14 | 2.1826 | - | - | 164.4311 | 115.9619 | 25.71 | 23.06 |
| | PSO | 2 | 1.1141 | 14 | 2.6882 | 3 | 0.7912 | 166.1743 | 117.1713 | 24.53 | 22.03 |
| | ICA | 17 | 1.2906 | - | - | - | - | 158.7471 | 116.4881 | 28.3 | 22.43 |
| | ICA | 13 | 1.2352 | 17 | 1.1193 | - | - | 158.3501 | 117.1534 | 28.48 | 21.99 |
| | ICA | 11 | 0.9617 | 9 | 1.0046 | 17 | 1.5319 | 158.0996 | 118.2645 | 28.6 | 21.25 |

In 33-bus systems, in Tables 2, by applying ICA to this system the same optimal size and location is obtained as well as heuristic search and PSO algorithm

***Conclusion:***

new evolutionary algorithm known as ICA is discussed to solve the OPDG problem. ICA for optimal placement of multi-DGs is to efficiently minimize total real power loss and voltage improvement, while satisfying transmission line limits and constraints. Applying This method results in fast and accurate in sizes and locations determination. This method is compared to the other methods. The simulation results show that, computation times and Loss of ICA are less than other algorithms such as Heuristic Search and PSO.

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