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Buckling behavior of Optimal Laminated Composite Cylindrical Shells Subjected to axial compression and external pressure

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Abstract. In this study, the buckling behavior of optimum laminated composite cylindrical shells subjected to axial compression and external pressure are studied. The cylindrical shells are composed of multi orthotropic layers that the principal axis gets along with the shell axis (x). The number of layers and the fiber orientation of layers are selected as optimization design variables with the aim to find the optimal laminated composite cylindrical shells. The optimization procedure was formulated with the objective of finding the highest buckling pressure. The Genetic Algorithm (GA) and Imperialist Competitive Algorithm (ICA) are two optimization algorithms that are used in this optimization procedure and the results were compared. Also, the effect of materials properties on buckling behavior was analyzed and studied.

Introduction

Laminated composite materials find a wide range of applications in structural design, especially in the field of automotive, aerospace and marine engineering. This wide range of application is mainly due to the high specific strength and stiffness values with minimum weight that these type of materials offer. Although composite materials are attractive replacement for metallic materials for many structural applications, the design and analysis of this kind of materials is more complex than those of metallic structures.

Laminated composite materials and structures find a wide range of applications in structural design, especially in the field of automotive, aerospace and marine engineering. This wide range of application is mainly due to the high specific strength and stiffness combined with minimum weight that these materials offer. Although composite materials are attractive replacement for metallic materials for many structural applications, the design and analysis of this kind of materials is more complex than those of metallic structures.

Various analysts have studied the optimization of laminated composite cylindrical shells subject to various loadings [1-3]. Khong [4] presented the optimal design of laminates for maximum buckling resistance and minimum weight. Alibeigloo [5] studied optimal stacking sequence of laminated anisotropic cylindrical panels using genetic algorithm to obtain the optimum design of composite laminates based on maximum buckling load. Topal [6] studied multiobjective optimization of laminated composite cylindrical shells for maximum frequency and buckling load. Damador and Navin [7] worked on optimal design of grid-stiffened panels and shells with variable curvatures. Akl and Ruzzene [8] carried out the optimum design of underwater isotropic stiffened cylindrical shells. Wang et al. [9] considered the cost and weight of laminated composite cylindrical shell as the optimization criteria, the combination of which is performed using Pareto method and the optimal solution is obtained from a curve extracted from the method. Walker [10] worked on multiobjective optimization of maximizing the weighted sum of the critical buckling load and the resonance frequency for a given laminated plate thickness by optimally determining the fiber orientations.

In the field of optimization, Atashpaz-Gargari and Lucas [11] introduced the Imperialist Competitive Algorithm (ICA). This algorithm is a new socio–politically motivated global search strategy that has recently been introduced for dealing with different optimization tasks. This evolutionary optimization strategy has shown great performance in both convergence rate and better global optima achievement [12].

The present study is devoted to using the ICA and Genetic algorithm to search for the optimum layers of laminated composite cylindrical shell under axial compression loading and external pressure. The number of layers and the orientation of fibers in each layer are selected as optimization design variables. The optimization procedure was formulated with the objective to find the highest buckling pressure per unit weight. Hence the buckling behavior of laminated composite cylindrical shell was initially studied. The results of the ICA are then compared with the results of genetic algorithm. Graphite/Epoxy and Glass/Epoxy composites were used in this study to analyze the effect of material on buckling behavior of laminated composite cylindrical shell under axial compression loading and external pressure.

Problem statement

Consider a laminated composite cylindrical shell composed of n orthotropic layers, shown in Figure 1, having mean radius R, wall thickness h, and length L, under compressive axial compression loading and then external pressure. The ends of the cylindrical shell are supported by rings which are rigid in their planes, but offer no resistance to rotation or displacement out of their planes. Prebuckled deformations are not taken into account. The laminated composite cylindrical shell will be buckled under compressive axial loading when the axial loading reaches the critical buckling load [13]:

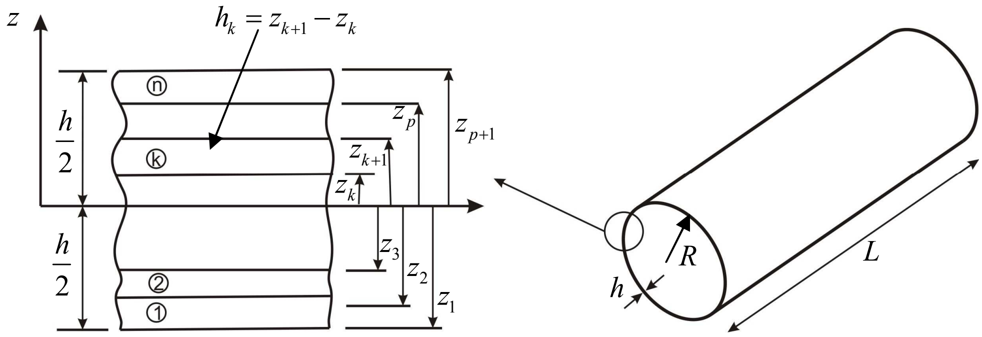

Figure 1: Laminated composite cylindrical shell

$$N_{x_{cr}} = \left(\frac{L}{m\pi}\right)^2 \frac{\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}}{\begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix}} \tag{1}$$

where, $N_{x_{cr}}$ is the critical compressive load, L and R are the length and radius of cylindrical shell, respectively. m is the number of buckled half waves in the axial direction and n is the number of buckled half waves in the circumferential direction. The coefficients C, A, B and D are defined as follows [13]

$$C_{11} = A_{11}\left(\frac{m\pi}{L}\right)^2 + A_{66}\left(\frac{n}{R}\right)^2 \tag{2}$$

$$C_{22} = A_{22}\left(\frac{n}{R}\right)^2 + A_{66}\left(\frac{m\pi}{L}\right)^2 \tag{3}$$

$$C_{33} = D_{11}\left(\frac{m\pi}{L}\right)^4 + \left(4D_{66} + 2D_{12}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{n}{R}\right)^2 + D_{22}\left(\frac{n}{R}\right)^4 + \frac{A_{22}}{R^2} + \frac{2B_{22}}{R}\left(\frac{n}{R}\right)^2 + \frac{2B_{12}}{R}\left(\frac{m\pi}{L}\right)^2 \quad (4)$$

$$C_{12} = C_{21} = \left(A_{12} + A_{66}\right)\left(\frac{m\pi}{L}\right)\left(\frac{n}{R}\right) \quad (5)$$

$$C_{23} = C_{32} = \left(B_{12} + 2B_{66}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{n}{R}\right) + \frac{A_{22}}{R}\left(\frac{n}{R}\right) + B_{22}\left(\frac{n}{R}\right)^3 \quad (6)$$

$$C_{13} = C_{31} = \frac{A_{12}}{R}\left(\frac{m\pi}{L}\right) + B_{11}\left(\frac{m\pi}{L}\right)^3 + \left(B_{12} + 2B_{66}\right)\left(\frac{m\pi}{L}\right)\left(\frac{n}{R}\right)^2 \quad (7)$$

$$A_{ij} = \sum_{k=1}^{N}\left[\bar{Q}_{ij}\right]_k\left(h_k - h_{k-1}\right), \quad B_{ij} = \frac{1}{2}\sum_{k=1}^{N}\left[\bar{Q}_{ij}\right]_k\left(h_k^2 - h_{k-1}^2\right), \quad D_{ij} = \frac{1}{3}\sum_{k=1}^{N}\left[\bar{Q}_{ij}\right]_k\left(h_k^3 - h_{k-1}^3\right) \quad (8)$$

where, A_{ij}, B_{ij} and D_{ij} are the extensional stiffness matrix, the bending-extension coupling matrix and the flexural or bending stiffness matrix of the shell composite laminate structure.

After determining the critical load, a check must be made to see that the final construction is not overstressed at a load below the critical buckling load (what is the reason?). For a laminated composite structure, the most general constitutive equation for the cylindrical shell is given by:

$$\begin{bmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 2A_{16} & B_{11} & B_{12} & 2B_{16} \\ A_{12} & A_{22} & 2A_{26} & B_{12} & B_{22} & 2B_{26} \\ A_{16} & A_{26} & 2A_{66} & B_{16} & B_{26} & 2B_{66} \\ B_{11} & B_{12} & 2B_{16} & D_{11} & D_{12} & 2D_{16} \\ B_{12} & B_{22} & 2B_{26} & D_{12} & D_{22} & 2D_{26} \\ B_{16} & B_{26} & 2B_{66} & D_{16} & D_{26} & 2D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_\theta^0 \\ \varepsilon_{x\theta}^0 \\ \kappa_x \\ \kappa_\theta \\ \kappa_{x\theta} \end{bmatrix} \quad (9)$$

In the case of buckling due to the axial compression, $N_x = N_{x_{cr}}$, $N_\theta = N_{x\theta} = M_x = M_\theta = M_{x\theta} = 0$. After finding $\left[\varepsilon^0\right]$ and $\left[\kappa\right]$ matrices from the equation 9, each stress component $\sigma_x, \sigma_\theta, \sigma_{x\theta}$, in each lamina or ply can be calculated by [13]:

$$\begin{bmatrix} \sigma_x \\ \sigma_\theta \\ \sigma_{x\theta} \end{bmatrix}_k = \left[\bar{Q}\right]_k \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_\theta^0 \\ \varepsilon_{x\theta}^0 \end{bmatrix} + z\left[\bar{Q}\right]_k \begin{bmatrix} \kappa_x \\ \kappa_\theta \\ \kappa_{x\theta} \end{bmatrix} \quad (10)$$

These stresses can then be compared to the allowable or failure stress in each ply [13].

In the case of cylindrical shell under external pressure, the critical value of pressure, p_{cr}, that will cause buckling is determined by [13]:

$$p_{cr} = \frac{R}{n^2} \frac{\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}}{\begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix}} \quad (11)$$

where, C_{ij} are defined by Equations 2 through 10.

For the external pressure only, $N_\sigma = p_{cr}R,\quad N_x = N_{x\theta} = M_x = M_\theta = M_{x\theta} = 0$ As in axial compression, checks are made to be ensure that overstressing does not occur at a pressure lower than the critical pressure, by using Equation 9 and Equation 10 [13].

Imperialist Competitive Algorithm

The Imperialist Competitive Algorithm (ICA) [11] is a numerical method that is used to solve many types of optimization problems. Like most of the methods in the field of evolutionary computation, ICA does not need the gradient of the function in its optimization process.

Specifically, ICA can be thought of as the social counterpart of genetic algorithms (GAs). ICA is the mathematical model and simulation of human social evolution, while GAs is based on the biological evolution of species.

This algorithm starts with an initial population. Each individual of the population is called a country. Countries are the counterpart of chromosomes in GAs. After evaluating the cost function of each country, some of the best of them (in optimization terminology, countries with the least cost) are selected to form the initial empires by controlling the other countries (colonies). All the colonies are divided among the initial imperialists based on their power. The power of each country, the counterpart of fitness value in the GA, is inversely proportional to its cost. The initial imperialist states together with their colonies form the initial empires.

After forming initial empires, the evolution begins. The colonies in each empire start moving toward their relevant imperialist country. This movement is a simple model of assimilation policy which was pursued by some of the imperialist states. Besides assimilation, revolution is another operator of this algorithm. Revolution occurs in some of the colonies by making random changes to their position in the socio-political axis. The total power of an empire depends on both the power of the imperialist country and the power of its colonies.

Imperialistic Competition is another step of the algorithm. All empires try to take the possession of colonies of other empires and control them. The imperialistic competition gradually brings about a decrease in the power of weaker empires and an increase in the power of more powerful ones. The imperialistic competition is modeled by picking some (usually one) of the colonies of the weakest empire and making a competition among all empires to possess these (this) colonies.

The above steps continue until a stop condition is satisfied by reaching to an acceptable suboptimal solution [14-15].

Mathematical Formulation

A laminated composite cylindrical shell under axial compression loading and external pressure is optimized for the maximum critical buckling load satisfying specified design requirements. Ply orientation and number of plies of composite shell are the preassigned variable parameters for this design problem. The following optimization problem can then be formulated.

- **Cost Function**

Engineering demands for high critical buckling load with low weight are meant to save materials, to enhance shipping and erection procedures and reduce fabrication costs. Therefore, the cost function for critical buckling per unit weight is considered to be maximized in this paper,

$$\text{Maximize}\, F\left(\tilde{x}\right) = N_{x_{cr}} / W \tag{12}$$

where $N_{x_{cr}}$ is the critical buckling load of laminated composite cylindrical and W is the weight of shell.

- **Design Variables**

The ply orientation and the number of layers are selected in this study as design variables. Therefore,

The number of layers, $x_1 = n, \quad 2 \le n \le 16$

The angle of layers, $x_2 = \theta \quad -90 \le \theta \le 90^o$

Therefore, the vector $\tilde{x}$ will be given as

$$\tilde{x}_{opt} = \left(n, \theta\right) \tag{13}$$

Model description

Consider a laminated composite cylindrical shell made of multi orthotropic layers that the principal axis (x) gets along with the shell axis. Graphite/Epoxy and Glass/Epoxy composites are used in this study and the mechanical properties are listed in Table 1. The dimensions of the cylinder used in this study are similar to those of Refs [16], viz, 250 mm mean radius, 2500 mm length and a total thickness of 5 mm.

Table 1: Mechanical properties of a unidirectional lamina

Property	Symbol	Unit	Glass/Epoxy	Graphite/Epoxy
Longitudinal elastic modulus	E_1	*GPa*	38.6	181
Transverse elastic modulus	E_2	*GPa*	8.27	10.30
Major Poisson's ratio	v_{12}	---	0.26	0.28
Shear modulus	G_{12}	*GPa*	4.14	7.17

Numerical analysis

The aim of the analysis is to find the optimum number of layers and the orientation of layer fibers of a laminated composite cylindrical shell under axial compression loading and external pressure which will result in maximum critical buckling load. In this study two material types, graphite/epoxy and glass/epoxy, were used after finding the optimum orientation and number of layers, and the buckling behavior of shell was studied. Table 2 and Table 3 show the optimum ply orientation of composite cylindrical shell under axial compression loading for some specific number of layers analyzed for both types of materials. The designs were optimized using imperialist competitive algorithm.

Table 2: Optimum plies orientation of laminated composite cylindrical shell under axial loading

Number of Layers	Optimal ply orientation													
n=4	-16.0	-48.5	9.3	-8.2								Graphite/Epoxy		
n=6	-19.4	-5.9	5.1	35.6	-20.3	-60.0								
n=8	74.9	-13.4	60.3	1.1	30.6	6.6	-20.0	87.2						
n=10	12.6	-1.5	13.3	54.7	-59.7	-44.7	15.5	-21.6	85.1	66.4				
n=12	-0.6	-16.9	33.0	35.0	21.1	-67.2	8.8	90.0	-61.8	-27.6	84.6	-73.9		
n=14	-2.8	15.8	34.5	21.1	-63.2	21.9	-82.7	-20.8	47.7	-14.6	27.2	87.7	76.9	-72.3

Also, Table 4 and Table 5 show the optimum ply orientation of composite cylindrical shell under external pressure for both types of materials, optimized using imperialist competitive algorithm.

Table 3: Optimum plies orientation of laminated composite cylindrical shell under axial loading

Number of Layers	Optimal ply orientation													
n=4	15.6	22.1	-41.7	-8.4								Glass/Epoxy		
n=6	63.0	-21.9	70.4	-22.6	-9.9	52.8								
n=8	-52.3	-73.6	8.7	29.7	19.2	16.0	10.4	-29.3						
n=10	23.9	-44.6	22.2	-38.5	25.0	26.5	7.5	44.7	-7.0	88.1				
n=12	6.6	-13.2	-28.9	77.8	-16.8	-16.8	38.1	-40.2	69.9	68.4	-7.8	-77.8		
n=14	25.9	3.6	-72.5	-14.5	19.3	45.1	13.6	-71.9	54.6	22.5	24.6	7.2	-81.2	73.6

Table 4: Optimum plies orientation of laminated composite cylindrical shell under external pressure

Number of Layers	Optimal ply orientation													
n=4	18.7	-41.2	14.7	31.1								Graphite/Epoxy		
n=6	25.8	-4.9	-16.0	-15.4	31.0	-12.9								
n=8	-33.4	-27.0	-15.5	6.7	-35.4	-11.2	-65.5	-48.8						
n=10	-47.6	12.6	3.7	27.9	-6.7	-33.2	15.7	34.5	80.2	-59.9				
n=12	-4.9	58.1	20.7	28.8	-26.3	-28.6	7.4	-21.6	54.1	-48.6	-79.8	-58.5		
n=14	-8.0	-29.8	49.1	0.0	-10.8	57.5	18.0	2.8	39.8	56.2	-25.6	36.0	-74.9	-59.0

Table 5: Optimum plies orientation of laminated composite cylindrical shell under external pressure

Number of Layers	Optimal ply orientation													
n=4	-24.9	-17.6	-20.4	0.6								Glass/Epoxy		
n=6	-3.5	8.4	25.3	1.4	9.7	36.9								
n=8	38.0	27.8	33.4	-30.3	3.9	23.2	7.8	-38.6						
n=10	-24.5	36.5	-23.3	-9.0	17.9	-6.4	8.7	24.6	-69.4	78.0				
n=12	-23.8	-7.6	19.1	1.5	-5.6	-29.1	-33.5	57.9	45.5	16.1	81.2	-51.0		
n=14	-20.6	33.7	57.9	37.0	-16.0	-13.2	-18.9	49.0	3.1	-9.6	-22.3	-78.4	-80.2	54.4

The effect of the number of optimum layers on the critical buckling load of laminated composite cylindrical shell made of Graphite/Epoxy and Glass/Epoxy under axial compression loading and external pressure by using ICA is shown in Figure 2.
The GA and ICA are two optimization algorithms that are used in this study to find the optimum laminated composite cylindrical shell under axial compression loading and external pressure. The critical buckling behavior of cylindrical shells composed of Glass/epoxy by using GA and ICA under axial compression loading and external pressure were studied and the results of both algorithms was compared. The results for axial compression loading are shown in Figure 3.

From Figure 2, the effect of optimum number of layers on critical buckling load of shell composed of graphite/epoxy or glass/ epoxy under axial compression loading are more than external pressure and laminated composite cylindrical shells composed of graphite/epoxy are four times stronger than the cylindrical shell composed of glass/epoxy under axial compression loading.

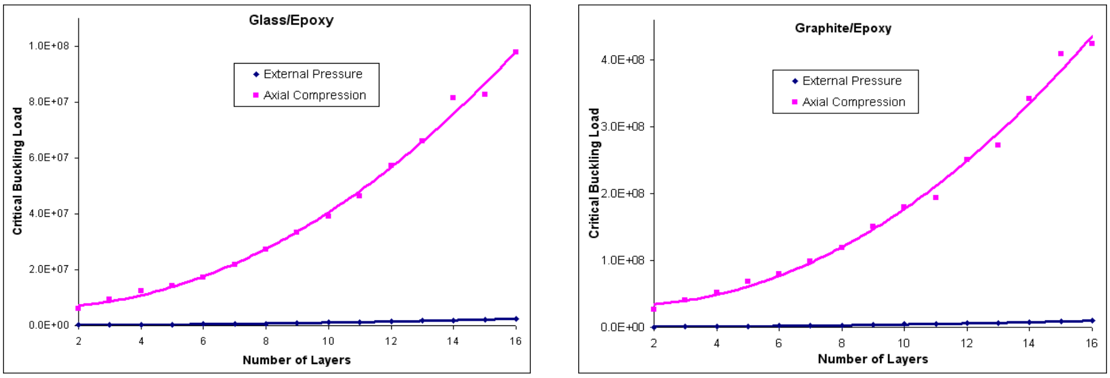

Figure 2: Effect of optimum number of layers and materials type on critical buckling behavior of composite cylindrical shell under axial compression loading and external pressure, using ICA.

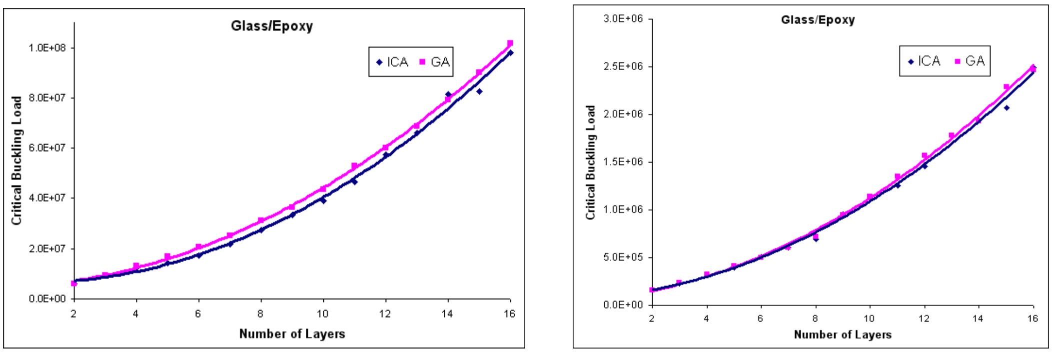

Figure 3: Figure 2: Effect of optimum number of layers on critical buckling behavior of composite cylindrical shell under axial compression loading and external pressure, using ICA and GA.

From Figure 3, the performance of ICA and GA are almost are same. It can be seen that the critical buckling load of laminated composite cylindrical shells made of Glass/epoxy under axial compression loading is higher than cylindrical shells under external pressure.

Conclusion

The Imperialist competitive algorithm (ICA) and genetic algorithm (GA) are used in this optimization problem to find the optimum number of layers of laminated cylindrical shell under various loadings. It can seen that the imperialist competitive algorithm is a very efficient and easy to use algorithm for optimization problems such as the optimization of laminated composite plate, cylindrical shell and panel. The results of imperialist competitive algorithm are very close to the results of genetic algorithm. The ICA has been shown to be used effectively in discrete optimization problems and multiobjective optimization problems like the present optimization exercise.
In this study, the effect of increasing the number of layers on buckling behavior of laminated composite cylindrical shell composed of graphite/epoxy and glass/epoxy under two types of loading was studied.

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