## Optimization of Temperature-Dependent Functionally Graded Material Based on Colonial Competitive Algorithm

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**Abstract.** A functionally graded material is optimized for thermal buckling. The buckling problem is formulated where temperature-dependence constituents' properties are included. Then the Colonial Competitive Algorithm (CCA) is used for optimization of critical thermal buckling load with variation thickness. The results CCA optimization is compared with third order shear deformation theory and another optimum method based on Genetic Algorithm (GA).

#### Introduction

Functionally Graded Materials (FGMs) are composite materials developed to resist ultimate high temperature conditions. The main advantage of such materials is the possibility of tailoring desired properties to needs. Obviously, FGMs can be used in a variety of applications which have made them very attractive.

Mozafari [1, 2] has studied the mechanical and thermal buckling behavior of FGM plates with variation thickness. FGM optimization problems are investigated using neural network methods to optimize the composition of plates and cylinders with grad thickness [3, 4] and using genetic algorithms to optimize stacking sequence in laminated plates [5].

There are several CCA optimization studies carried out in recent years. Mozafari [6, 7] used the colonial or the imperialist competitive algorithm (CCA or ICA) to optimize thin interphase layer in composite materials. Mozafari [8] used ICA to optimize composite. Abdi [9, 10] optimized end domes under buckling pressure, based on CCA and GA.

The aim of this paper is optimization of critical thermal buckling load for a plate made of functionally graded materials, using CCA and to compare the CCA results to GA. The comparison shows the success of CCA for optimizing FGM plates.

#### **Problem Formulation**

Consider a functionally graded thin plate made from a mixture of ceramics and metals and subjected to a thermal load. The plate coordinate system (x, y, z) is chosen such that, x and y are inplane coordinates and z is in the direction through the plate thickness. The corresponding displacements in the x-, y- and z-directions are designated by u, v and w, respectively. (See Fig. 1).

$$h = h(x) = \xi = c_1 x + c_2$$
 and  $h = h(y) = \xi = c_1 y + c_2$ . (1)

in which  $\xi$  is a general parameter indicating the thickness change in either of x or y directions,  $c_2$  is the nominal thickness of the plate at the origin and  $c_1$  is a variable non-dimensional parameter. When  $c_1$ =0, it means that the plate has a constant thickness. When x = 0, one has h =  $\xi$  =  $c_2$  and for the case of x = a,  $\xi$  (a) =  $c_1 a$ + $c_2$ 

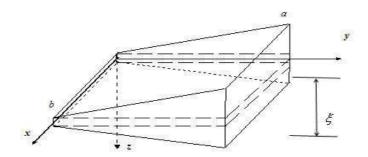


Figure 1: Illustration of geometry and coordinate system of FG plate

The higher order plate displacement theory which is considered in the present work is based on the assumption of the displacement field in the following form:

$$u(x, y, z) = u_0(x, y) - zw_{0,x},$$

$$v(x, y, z) = v_0(x, y) - zw_{0,y},$$

$$w(x, y, z) = w_0(x, y)$$
(2)

in which u, v, w are the total displacements and  $(u_0, v_0, w_0)$  are the mid-plane displacements in the x, y and z directions, respectively.

For a functionally graded plate, usually the temperature change in the structure is not uniform where the temperature level is much higher at the ceramic side than that in the metal side of the plate. In this case, the temperature variation through the thickness is given by;

$$T(z) = \frac{\Delta T_{cr}}{\xi} \left( z + \frac{\xi}{2} \right) + T_m \tag{3}$$

in which,

$$T\bigg|_{z=\frac{\xi}{2}}=T_c\,,$$

$$T\bigg|_{z=-\frac{\xi}{2}} = T_m,\tag{4}$$

$$\Delta T_{cr} = T_c - T_m$$

 $T_c$  and  $T_m$  denote the temperature level at the top ( ceramic side ) and the bottom ( metal side) surfaces, respectively. The pre-buckling forces now can be obtained by solving the membrane form of equilibrium equations, as:

$$N_{x}^{0} = -\frac{c_{1}a/2 + c_{2}}{1 - v} (\Delta T_{cr}G_{2} + T_{m}G_{1}),$$

$$N_{y}^{0} = -\left[c_{1}x + \frac{vc_{1}a}{2(1 - v)} + \frac{c_{2}}{1 - v}\right] (\Delta T_{cr}G_{2} + T_{m}G_{1}), \qquad N_{xy}^{0} = 0$$
(5)

in which

$$G_2 = [E_m \alpha_m / 2 + (E_m \alpha_{cm} + E_{cm} \alpha_m) / (k+2) + E_{cm} \alpha_{cm} / (2k+2)], \tag{6}$$

where subscripts m and c refer to the metal and ceramic constituents, respectively. The thermal buckling load for the functional graded plate with linear temperature variation across the thickness can be obtained as:

$$\Delta T_{cr} = \frac{\pi^2 [(b/a)^2 + 1]}{b^2 (1+v) h G_2} \left(\frac{A.C - B^2}{A}\right) - \frac{T_m G_1}{G_2}$$
(7)

where A, B and C are the constants. The equation detail can be referred to in a paper by Mozafari [2].

#### **Optimization Problem**

The Colonial Competitive Algorithm (CCA) is a new evolutionary optimization method which is inspired by imperialistic competition. Like other evolutionary algorithms, it starts with an initial population which is called country and is divided into two types of colonies and imperialists who together form empires. The imperialistic competition among these empires forms the proposed evolutionary algorithm. Imperialistic competition converges to a state in which there exists only one empire.

The total power of an empire depends on both the power of the imperialist country and the power of its colonies which is shown as:

$$C.C_n = cost function (imperialist_n) + \lambda mean \{cost (colonies of empires_n)\}$$
 (8)

This competition gradually brings about a decrease in the power of weaker empires and an increase in the power of more powerful ones. This is modeled by just picking some of the weakest colonies of the weakest empires and making a competition among all empires to possess these colonies. Figure 2 is shown a flowchart of the CCA.

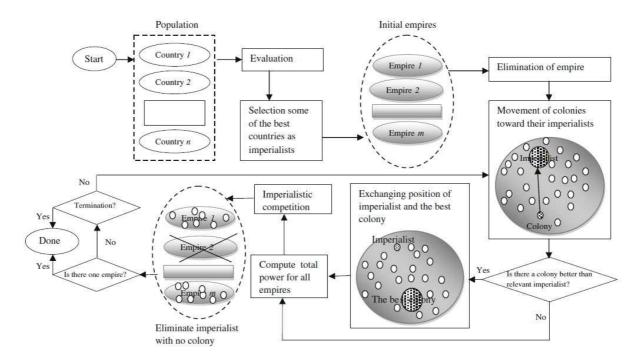


Figure 2: Illustration of optimization using CCA

The CCA optimization is used to investigate of the critical buckling temperature  $\Delta T_{cr}$  while maintaining the total structural mass constant at a value equals to that of a reference baseline design. The optimization variables include  $V_f$ , h,  $\theta$ , t, which are the fiber volume fraction, fiber orientation angle of the individual ply and thickness, respectively.

To obtain optimal design, the objective function is defined as follows,

$$X = (V_f, h_t^*, \theta_t) \qquad t = 1, 2, 3, ..., n \tag{9}$$

$$h_{t}^{*} = h_{t} / h \tag{10}$$

where  $h_t^*$  is the dimensionless thickness.

Then optimization is the maximization of the buckling temperature by changing the ply orientations:

Maximize 
$$\Delta T_{cr}^*$$

Subject to  $\sum_{t=1}^{N} V_f h_t^*$ 
 $V_t^- \leq V_f \leq V_t^+$ 
 $t = 1, 2, 3, ..., n$ 
 $h_t^- \leq h_t^* \leq h_t^+$ 
 $\theta_t^- \leq \theta_t \leq \theta_t^+$ 

(11)

The subscripts +, - denote the upper and lower bounds imposed on the various design variables. The dimensionless critical buckling load is:

$$\Delta T_{cr}^* = \Delta T_{cr} / \Delta T \tag{12}$$

In the above problem, there are *N* optimization parameters. It should be noted that if it is desired to minimize one or more objective functions, the duality principle states that a minimization problem can be converted to a maximization problem by multiplying the corresponding objective function values.

#### **Results and Discussion**

A functionally graded material consisting of aluminum and alumina is considered in which the Young's modulus, conductivity, and the coefficient of thermal expansion, are: for aluminum,  $K_m = 204w/mk$ ,  $\alpha_m = 23 \times 10^{-6} (1/^{0} C)$   $E_m = 70 GPa$  and for alumina,  $E_c = 380 GPa$ ,  $K_c = 10.4w/mk$ ,  $\alpha_c = 7.4 \times 10^{-6} (1/^{0} C)$ . The Poisson's ratio  $v_m = v_c = 0.3$  are taken for both.

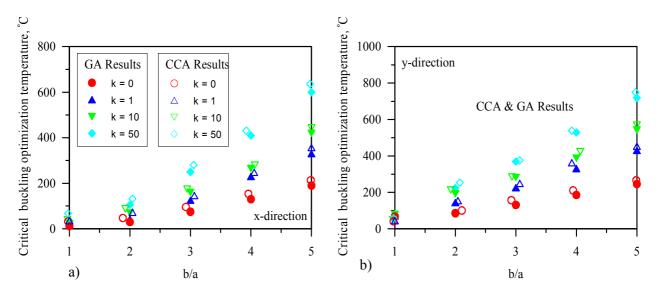


Figure 3: Effect of b/a on optimized buckling temperature of the functionally graded plate, from CCA and GA.

Figures 3a and 3b show the comparison between the CCA and GA optimized results of buckling temperature against material index k, from zero to 50 with respect to ratio b/a, with linear thickness change in both x and y-directions.

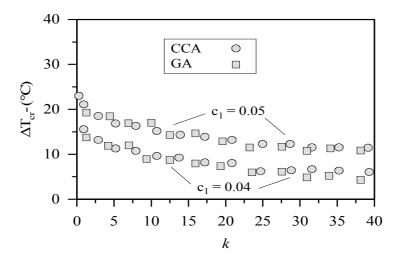


Figure 4: Effect of material graded index k on optimized buckling temperature - comparison between CCA and GA. b/a=1.

Figure 4 illustrates the CCA and GA results of the optimized critical buckling temperature with respect to the material graded index k. The optimum solution is obtained by setting CCA and GA coding in MATLAB program as the analyzer. The critical buckling temperatures are optimized under uniform temperature rise respect to k and b / a = 1. The differences between the optimized buckling temperatures are in Tables 1. The CCA results of the third order shear deformation theory (TSDT) are compared with the results obtained with genetic algorithm (GA). As can be seen the difference of DGT is not more than 10% and that of DCG is less than 0.5%.

Table 1: Critical buckling optimization temperature (°C) of the FG plate under uniform temperature rise, with respect to k and b/a = 1.

k	TSDT	Opt. GA	Opt. CCA	$^{9}D_{GT}$	$D_{CG}$
k = 0	46.853	51.504	51.737	9.03	0.45
k = 1	15.06	16.53	16.603	8.91	0.44
k = 10	13.018	14.184	14.24	8.22	0.39
k = 50	10.648	11.453	11.49	7.03	0.34

The values of the deviations of (GA) to (TSDT) and (CCA) to (GA) are shown by  $D_{GT}$  and  $D_{CT}$ , respectively.

#### **Conclusion**

In this paper, a functionally graded plate made of metal and ceramic is optimized based on a new optimization theory algorithm called the colonial competitive algorithm (CCA). The aim of this paper is to maximize the critical thermal buckling load of FG plates with linear temperature variation across the thickness.

The two methods CCA and GA were then compared with the third order shear deformation theory. Extremely high accuracy could be achieved over the whole range of the plate for a problem with linear temperature variation.

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