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Of late, attempts are being made to optimise production system problems by minimum cost. A good available device in this area is response surface methodology. This methodology combines experimental designs and statistical techniques for empirical model building and optimising. In most situations simulated models for real world problems are non-linear multi-response, while responses are conflicting. The simultaneous optimisation of several conflicting responses is computationally expensive. So this makes the problem solving extremely complex. Since few attempts have been made to scrutinise this domain, in this paper the nonlinear continuous multi-response problem is investigated. In order to tackle multi-response optimisation difficulties, we propose a new hybrid meta heuristic based on the imperialist competitive algorithm. It simulates a socio–economical procedure, imperialistic competition. When there are some non-dominated solutions in searching space, a technique for order performance by similarity to ideal solution is used to identify which non-dominated solutions are imperialists and which ones belong to colonial societies. A particle swarm-like mechanism is employed to model the influence of imperialists on colonies. The algorithm will continue until only one imperialist obtains all countries' possessions. In order to prevent carrying out extensive experiments to find optimum parameters of the algorithm, we apply the Taguchi approach. Since there is no standard benchmark in this field, we use three case studies from distinguished papers in the multi-response optimisation field. Comparing the results with some works mentioned in the literature reveals the superiority of the proposed algorithm.

Keywords: multi–response problem; imperialist competitive algorithm; MCDM; TOPSIS

1. Introduction

Nowadays there are varying products in markets and consumers lean towards those of better quality, with better after-sales service and of lesser price. Growing competition in markets forces manufacturers to improve the performance of their systems continuously. To augment sales and benefits, they have to enhance the quality of the products without increasing the cost.

One of the most important stages in a system's cost saving is simulating the manufacturing processes by mathematical relationships and optimising the model. This is named offline quality control. It can lead to minimising process variations, failure rates, reworks, scraps and the need for inspection and consequently increases the efficiency of the process, quality of end products and customer service level. Response surface methodology (RSM) is a common tool in this area that has been used in the last decades. In fact RSM is an incorporation of statistical and mathematical techniques, which is used to design and model the cause-and-effect relationship between process inputs (called effective variables or factors) and outputs (called responses).

There are two different significant groups of factors affecting the production process: controllable and uncontrollable. Uncontrollable factors, as the name suggests, can't be controlled within the production environment, though some of them may become under control during experimental conditions (see Figure 1). As mentioned above, RSM can generate a mathematical relationship between controllable input factors as independent input variables and the output of a process as dependent variables (or response variables).

Thus, if we succeed in justifying controllable factors we can increase the expected value of the product quality. So, it is interesting to find the impact of the effective controllable input factors, alone or in combination, in the processes.

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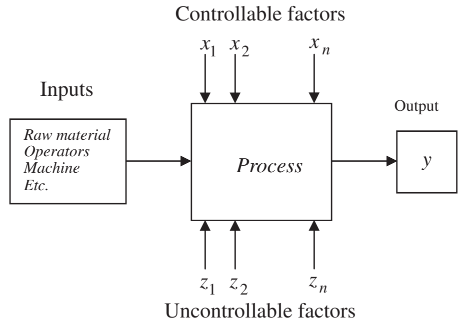

Figure 1. Production environment.

Normally, most of the real world engineering problems have several responses (or objectives). The simultaneous optimisation of several conflicting responses in order to find the solution(s) of a problem is known as a multi-response optimisation problem which can be computationally expensive and often makes the problem solving extremely complex.

Traditionally, there are several methods available in the literature for solving and optimising multi-response optimisation problems (MROPs) like goal attainment (Xu *et al.* 2004), goal programming (Clayton *et al.* 1982), the weighting method (Ilhan *et al.* 1992), the e-constraint method (Coello Coello 2000), and so on. But, none of these methods consider all the responses simultaneously, which is a basic requirement in most MROPs.

Comprehensive investigations have been carried out through different traditional techniques and are presented briefly below. For example, Myers and Carter (1973) used the bounded objectives method for the first time. Biles (1973) extended this method for more than two solutions. Myers *et al.* (1973) improved upon these results by combining them with the method of Myers and Carter. They used deviation and response effects as two independent solutions in their method. The decision function was proposed for the first time by Derringer and Suich (1980). The basic idea in this method was transforming multi-objective functions into a single objective. They converted each response function into a desirability function and maximised the geometric mean of the desirability of each response by using a single-objective optimisation technique. Zimmermann (1978) offered a method for solving multi-objective linear programming problems using fuzzy logic. Later Cheng *et al.* (2002) extended Zimmermann's method for optimising statistical multi-response problems but the solution's length was the weakness of their method.

Although RSM is a modelling tool, it is an optimisation technique too. But it is not applicable in complicated cases such as non-polynomials, higher-orders and multi-modal functions (Pasandideh and Niaki 2006). Recently, evolutionary algorithms (EAs) have been found to be appropriate for solving MROPs because they have some superiority to traditional techniques. For example, contrary to mathematical techniques, some special characteristics of functions like convexity, concavity, continuity and so on are not important in EAs. Thus, growing attention has been considered on utilising EAs for these problems in the last decade. For example, Correia *et al.* (2005) offered a comparison between RSM and genetic algorithm (GA) in the optimisation of welding processes. Suresh *et al.* (2002) presented a second level model for the expected roughness degree of steel parts. Their study tries to optimise the roughness degree of the parts by GA. Oktem *et al.* (2005) offered a GA method using RSM in which RSM presents an effective model to determine the level of parameters and GA optimises these levels. Fan *et al.* (2004) integrated the Nelder-Mead simplex search method with genetic algorithm and particle swarm optimisation (PSO) to locate the global optimum solutions for the nonlinear functions with continuous variable, mainly focusing on RSM. Ozcelik and Erzurumlu (2005) offered a model for warpage applying RSM. They developed a genetic algorithm for minimising the warpage on thin-shell plastic parts.

Pasandideh and Niaki (2006) modelled statistical multi-response optimisation problems using the desirability function approach and also proposed a GA with four different chromosome-selection strategies to solve it. Fourman (1985) suggested a GA-based method for the lexicographic ordering problem. In his approach, the decision maker ranks the objectives with respect to their importance. Then the optimum solution is obtained by optimising the objective function starting from the most important.

Kim *et al.* (2002) proposed a method based on the desirability function and GA to optimise a welding process. Khoo and Chen (2001) combined GA with RSM and also presented a GA in three different scenarios which can deal with single-response, multi-response and multi-constraint problems.

Analysing several qualitative attributes simultaneously, Koksoy and Yalcinoz (2006) used RSM and compared its results with those of the GA. A technique for solving multi-response problems in surface applications was presented by Ortiz *et al.* (2004). The technique combined an unconstrained desirability function with a genetic algorithm. Also it is capable of distinguishing between far from feasible and nearly feasible solutions. Fallah-Jamshidi *et al.* (2010) presented a novel two-phase hybrid genetic-based metaheuristic for nonlinear continuous multi-response problems.

EAs are inspired by events of nature and try to mimic the evolutionary process. An evolutionary algorithm keeps one or more populations of solutions for a given problem, and tries to improve upon these solutions by imitating evolutionary procedures. For example, a genetic algorithm is inspired by the biological principles of evolution; ant colony optimisation is inspired by the foraging behaviour of real ants; simulated annealing is inspired by an analogy to physical annealing in materials; and PSO is based on social interactions and communications such as birds flocking and fish schooling.

In contrast to the afore-mentioned algorithms that emulate natural behaviours and the biological evolution of humans or other living beings, recently a new EA has been proposed by Atashpaz-Gargari and Lucas (2008). This algorithm uses the socio–political evolution of humans as a source of inspiration for developing a powerful optimisation strategy. This method is named the imperialist competitive algorithm (ICA). By now ICA has been applied for single-objective problems. In our paper, we mainly introduce an adapted multi-objective ICA based on multi-criteria decision making (MCDM) theory, and utilise this algorithm to solve multi–response optimisation problems.

The rest of the paper is organised as follows. Section 2 describes the problem definition. In Section 3, the proposed algorithm is depicted. Computational experiments are explained in Section 4. In Section 5, experimental results are presented. Finally, Section 6 concludes the paper and introduces trends for future research.

2. Problem definition

Real world production system problems tackle several different responses. Events take place stochastically and various factors (independent input variables) affect multiple responses coincidentally. Therefore, the purpose of this research is to ascertain factor levels that in some senses optimise all responses or at least keep them in desired ranges.

In order to attain this goal, the first step is to estimate an appropriate response surface model for each of the responses (objectives) through computer simulation. The estimation is necessary because the mathematical relationship between controllable factors as independent input variables and outputs of processes as response variables is generally unknown. In this situation, independent input variables are denoted by $\vec{x} = (x_1, x_2, \ldots, x_n)$ and the kth response is shown by $f_k(\vec{x})$. The second step is generating and exploiting an efficient approach to optimise multi-response models given by the previous step simultaneously. So we are interested in adjusting levels of independent input variables such as $(x_1^*, x_2^*, \ldots, x_n^*)$ to solve problems with the following form:

$$\begin{aligned} &\textit{Maximise} \text{ or } \textit{Minimise}\ \vec{f}(\vec{x}) = \left[f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})\right] \\ &\text{subject to:}\ \ L_j \le x_j \le U_j \end{aligned}$$

where L_j and U_j are lower and upper bounds for jth independent input variable and $f_k : I\!R^n \rightarrow I\!R$, $k = 1, \ldots, nf$.

3. Proposed algorithm

ICA is a new global search algorithm that simulates the socio–political process of real world imperialistic competition.

Like other EAs, this algorithm starts with an initial population of solutions which are named countries. These countries are divided into two categories according to their power (response functions values): imperialists and colonies. Imperialists are some countries with higher power (more response values) in the population, and colonies are the remaining countries (with less response values) assigned to the imperialists based on the imperialists' power.

A set of one imperialist and its colonies is called an empire. An assimilation strategy and extending the reign of a government beyond its territory are the bases of this algorithm. The assimilation strategy in the real world is the attempt of an imperialist to abrogate the civilisations and cultures of its colonies and impose upon them compulsory ones. In the proposed algorithm this strategy is simulated through a mathematical relationship.

During transfer to an imperialist, a colony might achieve a position with a better response value than the imperialist's. In this case, the colony takes the imperialist position. Then, in the imperialistic competition procedure, all empires attempt to possess colonies of other empires according to their total power. The total power of an empire relates to both the power of the imperialist and the power of its colonies.

This competition gradually leads weak empires to fail to keep possession of their colonies and when an empire loses all of its colonies it collapses.

Ultimately when all empires except the most powerful one have been eliminated, all of the colonies are under the control of the same empire. This means that the algorithm has found the best solution (Karimi *et al.* 2011). In the following, we describe all steps of the adapted algorithm according to our problem in detail.

3.1 *Representation*

In an optimisation problem, the aim is to attain an optimal solution in terms of variables of the problem. In our mentioned problem, we consider each solution as a country.

In mathematical form, each country is a string of $1 \times n$ in which n is the number of independent variables in the range of (L_j, U_j) and the jth number in the string denotes the coded value of the jth input variable. From a historical–cultural point of view, the variables are countries' cultures, languages, economical rules and so forth (see Figure 2).

3.2 *Initial population*

A set of countries called initial population is vital for the algorithm to start. The size of the initial population would be given for the algorithm. Since the initial population has a significant effect on the performance of the algorithm and its speed in reaching to the final solutions, we applied a simulation approach to produce countries. The following notation has been used to generate the initial values of each independent variable:

x_{ij}	jth input variable of ith country; $i = 1, \ldots, m$; $j = 1, \ldots, n$
L_{ij}	lower limit for jth input variable of ith country
U_{ij}	upper limit for jth input variable of ith country
α	random number between 0 and 1
m	population size

$$x_{ij} = U_{ij} - (U_{ij} - L_{ij}) \times \alpha$$

If α has a value of 0, it means the relevant variable is in its upper bound and if it has a value of 1, the relevant variable is in its lower bound.

3.3 *Evaluation of authority*

All countries should be evaluated through response functions. The evaluation allocates a value according to each response function to each country which discriminates against them. Assuming a minimisation problem, solutions which have lower values are better.

x_1	x_2	x_3	…	x_n
$L_1 \leq x_1 \leq U_1$	$L_2 \leq x_2 \leq U_2$	$L_3 \leq x_3 \leq U_3$	…	$L_n \leq x_n \leq U_n$

Figure 2. Representation of a country.

3.4 *Generation of initial empires*

Here a specific number of best countries (N_{imp}) with regards to response function values are picked out to be the imperialist states. All the remaining countries constitute colonies (N_{col}) that are divided among these imperialists based on the total power of the imperialists.

In multi-response problems, usually there are non-dominated solutions in the searching space. For better interpretation, consider the following two definitions in a minimising problem:

(I) Dominant solution: A vector $\vec{a} = (a_1, a_2, \ldots, a_n)$ dominates $\vec{b} = (b_1, b_2, \ldots, b_n)$ if and only if $\vec{a}$ is partially less than $\vec{b}$, for example:

$$\forall j \in \{1, 2, \ldots, n\}, \quad a_j \leq b_j \wedge \ni j \in \{1, 2, \ldots, n\}, \quad a_j < b_j.$$

(II) Non-dominated solution: A vector of decision variables $\vec{a} \in A \subset IR^n$ is non-dominated with respect to A if it is not dominated by any other solutions, that is to say there is not any $\vec{a}' \in A$ that $\vec{f}(\vec{a}') < \vec{f}(\vec{a})$ (Zandieh and Karimi 2010).

Since non-dominated solutions in the searching space might be more than N_{imp}, we need a method to discriminate against them. In other words, we want to clarify which ones among these non-dominated solutions should be considered as emperors and which ones as colonies. So we face a decision-making problem.

A decision-making problem is the process of finding the best choice from all feasible alternatives. Often multiplicity of criteria for judging in such problems is pervasive. In other words for many such problems, the decision maker wants to solve a MCDM problem.

3.4.1 *TOPSIS*

The technique for order performance by similarity to ideal solution (TOPSIS) is a well-known classical MCDM method with cardinal information, a ratio scale, on the criteria/attributes that was initiated by Hwang and Yoon (1981). This technique is based on the concept by which the chosen alternatives should have the shortest distance from the positive ideal solution (the best solution) and the farthest from the negative ideal solution (the worst solution). A TOPSIS solution is defined as the alternative which is simultaneously farthest from the negative ideal and closest to the ideal alternative.

According to the simulation comparison of Zanakis *et al*. (1998), TOPSIS has the fewest rank reversals among the eight methods of MCDM. Thus, TOPSIS has been chosen as the target technique for our selection problem. This method generally consists of the following steps:

Step 1: Obtain a decision matrix, where a set of possible alternatives $(x_1, x_2, \ldots, x_m)$ is compared to a set of criterion functions (objective functions) $(y_1, y_2, \ldots, y_m)$. An element a_{ij} of the matrix is a value indicating the performance rating of alternative x_i with regard to the criterion y_j, w_j is the weight of criterion y_j.

$$M = \begin{array}{c c} & \begin{array}{cccc} y_1 & y_2 & \cdots & y_n \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_m \end{array} & \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \end{array}$$

$$W_0 = \begin{bmatrix} w_1 & w_2 & \cdots & w_n \end{bmatrix}$$

$$\sum_{j=1}^{n} w_j = 1$$

Step 2: Convert raw values a_{ij} to normalised values nr_{ij} as:

$$nr_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}} \quad j = 1, 2, \ldots, n$$

Step 3: Calculate the weighted normalised values of decision matrix as:

$$W = w_j \times nr_{ij}$$

where w_j is the weight of the jth criterion set by the decision maker (DM)

Step 4: Identify the positive (A^+) and the negative (A^-) ideal solution.

$$\left(W_1^+, W_2^+, \ldots, W_n^+\right) = \left\{\left(\max_i W_{ij} | j \in J\right) \& \left(\min_i W_{ij} | j \in J'\right) | i = 1, 2, \ldots, m\right\}$$
$$\left(W_1^-, W_2^-, \ldots, W_n^-\right) = \left\{\left(\min_i W_{ij} | j \in J\right) \& \left(\max_i W_{ij} | j \in J'\right) | i = 1, 2, \ldots, m\right\}$$

Where $J = 1, 2, \ldots, n$ is a set of indexes of benefit criteria and $J' = 1, 2, \ldots, n$ is a set of indexes of cost criteria.

Step 5: Calculate the Euclidean distances for each solution from the positive and the negative ideal solutions.

$$d_i^+ = \sqrt{\sum_{j=1}^{n} \left(W_{ij} - W_j^+\right)^2} \quad i = 1, 2, \ldots, m$$

$$d_i^- = \sqrt{\sum_{j=1}^{n} \left(W_{ij} - W_j^-\right)^2} \quad i = 1, 2, \ldots, m$$

d_i^+ is the distance of ith alternative from the positive ideal solution
d_i^- is the distance of ith alternative from the negative ideal solution

Step 6: Calculate relative closeness to ideal solution for each solution.

$$cl_i = \frac{d_i^+}{d_i^+ + d_i^-} \quad 0 \leq cl_i \leq 1$$

Step 7: Sort the solutions in terms of similarity cl_i, from the most to the least similar. The solution the least close has best rank.

Finally, after Step 7, a number of countries that have the best ranking with regard to the predetermined number of empires are selected. Each imperialist, depending upon its power, has authority over a number of countries. To estimate the number of colonies that belong to each imperialist, firstly the relative power of each imperialist should be calculated as follows:

$$p_i = \left\lfloor \frac{cl_i}{\sum_{i=1}^{N_{\text{imp}}} cl_i} \right\rfloor$$

As mentioned earlier, cl_i is the relative closeness to the ideal solution for each solution. Now the number of colonies of each imperialist can be calculated as follows:

$$NC_i = round\left\{p_i \times N_{\text{col}}\right\}$$

The initial number of colonies of the ith empire is NC_i, which are selected randomly. N_{col} is the total number of colonies (that is to say the difference between the number of solutions and the number of predetermined imperialists).

Thus each imperialist together with its NC_i number of colonies creates an empire (Karimi *et al.* 2011).

3.5 *Assimilation strategy: movement of colonies toward the imperialist*

As mentioned previously, imperialists try to influence their colonies in different socio–political aspects. Although each imperialist attempts to absorb its colonies and make them a part of it, colonies may resist these changes and make some deviations. These deviations may be caused by the effect of other empires' assimilation strategies. In order to model this strategy, we adapt a PSO-like mechanism proposed by Fallah-Jamshidi *et al.* (2010).

This method simulates social interaction and communication such as birds flocking and fish schooling to improve colonies towards the optimum, which is the significant feature of this algorithm. This approach is implemented on colonies of each empire, so the movement is in the direction of the imperialist but it is not a direct movement because of the attraction–repulsion of the best imperialist.

Each colony adjusts its movement through the searching space by combining some aspect of its own and experiences of other imperialists with some random perturbations. The following is the mathematical representation of these concepts.

$$V_{id} = \frac{\left(\varphi_1 \times [p_{id} - x_{id}] + \varphi_2 \times \left[p_{gd} - x_{id}\right]\right)}{\chi}$$

where:

V_{id}	movement length of ith country in dth variable
χ	constriction factor
φ_1	cognitive power
p_{id}	current status of imperialist of ith colony in dth variable
x_{id}	current status of ith colony in dth variable
φ_2	social power
p_{gd}	current status of the best one among other imperialists in dth variable

3.5.1 *Constriction factor*

This factor is provided to avoid exploding and growing out of the bounds of movement length. It prevents producing infeasible solutions. We use a dynamic stochastic approach to set a constriction coefficient as follows:

$$\chi = \begin{cases} \chi_{\max} + \dfrac{\chi_{\max} - \chi_{\min}}{MI} \times CI & \text{if } rand(0,1) \geq 0.5 \\ -\left(\chi_{\max} + \dfrac{\chi_{\max} - \chi_{\min}}{MI} \times CI\right) & \text{if } rand(0,1) < 0.5 \end{cases}$$

where:

$$\chi_{\max}: \ \max\left([p_{id} - x_{id}], \left[p_{gd} - x_{id}\right]\right)$$
$$\chi_{\min}: \ \min\left([p_{id} - x_{id}], \left[p_{gd} - x_{id}\right]\right)$$

MI	maximum iteration
CI	current iteration

3.5.2 *Acceleration coefficients*

φ_1 and φ_2 determine the impressibility degree of power by imperialist of corresponding colonies and the power of other imperialists, respectively. When φ_1 is greater than φ_2, the colony tends towards its imperialist rather than other imperialists, and if φ_1 is smaller than φ_2, the colony trusts in imperialists other than its own. We applied the following heuristic technique to define acceleration factors:

$$\varphi_2 = 1 - (\rho)^{CI}$$
$$\varphi_1 = 1 - \varphi_2$$

where ρ is a constant number in the range of $(0, 1)$ and here we assume its value equal to 0.9.

3.5.3 *Next position*

The position of the ith colony in the dth variable is now changed by adding calculated movement length to the current position as follows:

$$X_{id} = x_{id} + V_{id}$$

3.6 *Exchanging positions of imperialist and colony*

In the movement toward the imperialist, a colony may get to a position with a better response value than its imperialist. In other words, the colony becomes so powerful that it can leave imperialism. In such a case, the colony takes the position of its imperialist and the imperialist transmutes into a colony (Karimi *et al.* 2011). The algorithm will continue by the new imperialist in a new position and then colonies start trying to seize this position. Figure 3(a) and Figure 3(b) depict the position exchange between a colony and the imperialist.

3.7 *Imperialistic competition*

As mentioned earlier, imperialism is the policy of extending the power and reign of a government beyond its own territories. In the imperialistic competition, imperialist states compete strongly to increase the number of their colonies and extend their empires by possessing more colonies from weaker empires.

Power or weakness of an empire is defined according to its total power. The total power of an empire depends on both the power of the imperialist and the power of its colonies. Based on total power, each empire has the probability of taking possession of the weakest colony of the weakest empire. In other words this colony will not be possessed by the most powerful empire, but this empire has more probability of possessing the mentioned colony.

The imperialistic competition is modelled by the transfer of the weakest colony of the weakest empire. This competition gradually culminates to a development in the power of a great empire and a decrease in the power of weaker ones. An empire will collapse if not able to succeed in imperialistic competition and so will lose all of its colonies. The total power of an empire is defined as follows:

$$TP_i = cl_i\big(\text{imperialist}_i\big) + \varepsilon \times \text{mean}\big\{cl_i(\text{colonies of empire}_i)\big\}$$

where TP_i is the total power of the ith empire and ε is a positive number between (0, 1) and near to 0. An increase in the value of ε will result in a more significant role for colonies in determining the total power of an empire. Now we can calculate the probability of possession of empires (Karimi *et al.* 2011); see Figure 4.

$$P_i = \left\lfloor \frac{TC_i}{\sum_{i=1}^{N_{\text{imp}}} TC_i} \right\rfloor$$

These values are shown in a vector to divide colonies among empires:

$$P = \left[P_1, P_2, P_3, \ldots, P_{N_{\text{imp}}}\right]$$

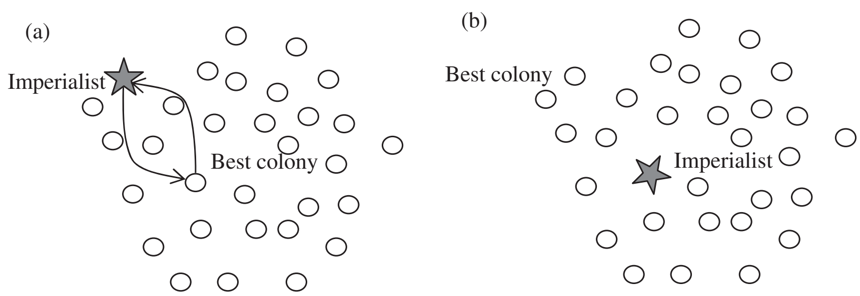

Figure 3. (a) Exchanging the positions of a colony and an imperialist. (b) New empire after position exchange.

To assign the weakest colony of the weakest empire, we need another vector with the same size that is generated by uniformly distributed random numbers, U(0, 1).

$$R = \left[r_1, r_2, r_3 \ldots, r_{N_{\text{imp}}}\right]$$

Now simply by subtracting R from P, the biggest value of vector D indicates which empire can possesses the weakest colony.

$$D = [P - R] = \left[P_1 - r_1, P_2 - r_2, P_3 - r_3, \ldots, P_{N_{\text{imp}}} - r_{N_{\text{imp}}}\right]$$

3.8 *Eliminating the powerless empires*

In the proposed algorithm, all empires fight for perpetuity by possessing more colonies from each other.

As the algorithm proceeds, any empire that is not able to increase its colonies or at least avoid decreasing its colonies will be very weak. Such empires have little chance of surviving. Each empire that loses all of its colonies will be eliminated (Karimi *et al*. 2011).

3.9 *Stopping criteria*

The algorithm stops when one or all of its stopping criteria are met. Here we consider the algorithm stops and the imperialistic competition terminates when only one empire remains. In this case, all the empires except the most powerful one are eliminated and all the colonies come under the authority of the last empire (Karimi *et al*. 2011).

4. Computational experiments

4.1 *Taguchi experimental design*

There are always several parameters that have remarkable influence on the performance of a metaheuristic algorithm. Parameter tuning is so important because different values of the effective parameters leads to different solutions. It is also difficult due to multimodality and nonlinearity of different kinds of response functions, especially in large-sized problems. These parameters are adjustable through trial and error or utilising different statistical parameter setting approaches like full factorial experiment. This is a comprehensive and the most widely used approach but loses its competence increasingly when the number of parameters is significantly high (Montgomery 2000).

Another technique to calibrate the algorithm is the Taguchi method, which reduces the number of experiments noticeably but gives sufficient information.

In the Taguchi approach, parameters are divided into two main groups: controllable and noise (uncontrollable) factors. Noise factors are those over which there is no direct control.

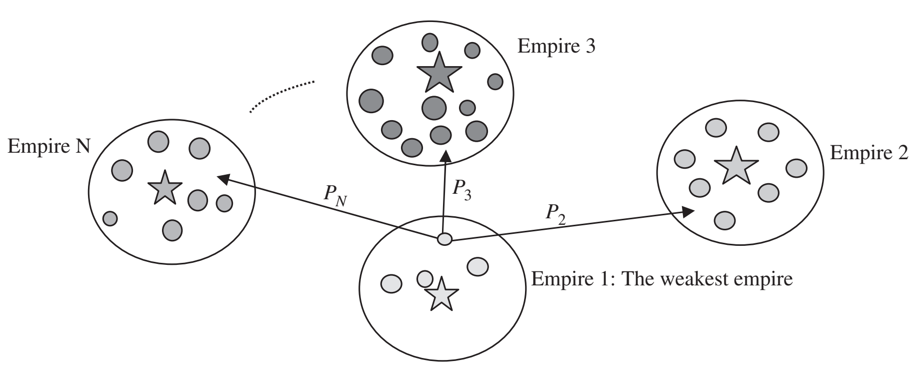

Figure 4. Imperialistic competition – Possession of the weakest colony of the weakest empire based on probability of possession.

We use the Taguchi method to find the optimal level of effective controllable factors while trying to minimise the effect of noise factors.

There are two major tools in this approach: the orthogonal array (OA) and the signal-to-noise (S/N) ratio. Terms "signal" and "noise" denote the mean response variable (desirable values) and the standard deviation (undesirable values), respectively.

An orthogonal array is a fractional factorial matrix that utilises scrutinising a large number of decision variables by a small number of experiments, and the S/N ratio facilitates specifying the amount of variation in the response variable. According to this method, controllable factors are located in the inner orthogonal array and noise factors are situated in the outer orthogonal array. The measured values that are obtained through the experiments will be transformed into an S/N ratio.

In the Taguchi approach, depending on the type of characteristics (continuous or discrete), S/N ratios are classified into three groups: nominal-is-the-best, smaller-the-better and larger-the-better. A detailed description can be found in Ross (1989) and Taguchi *et al.* (2000). Based on our problem features, we apply the smaller-the-better:

$$S/N \text{ ratio} = -10\log_{10}(\text{objective function})^2$$

4.2 *Parameter tuning*

Controllable factors of our algorithm are: population size, number of empires and the value of ε for calculating total cost. Factors with their levels are shown in Table 1. The fittest design for this algorithm is L_9 (3^3) as shown in Table 2.

We implemented these experiments in MATLAB 7.0.4 and ran them on a PC with 2.33 GHz Intel Core 2 Duo and 2 GB of RAM memory.

4.3 *Evaluation metric*

The conflicting nature of MROP solutions make us use some performance measures to ensure a better assessment of the proposed algorithm. Therefore we take three performance metrics into consideration. To start, we have to normalise responses, because they generally have different measurement units. Normalisation is accomplished for each response as follows:

$$Nf_k = \frac{f_k}{\max(f_k)} \text{ in a maximisation problem}$$

$$Nf_k = \frac{\max(f_k) - f_k}{\max(f_k) - \min(f_k)} \text{ in a minimisation problem}$$

where f_k and Nf_k are the kth response function and the normalised value of the kth response function, respectively.

Table 1. Factors and their levels.

	Controllable factors		
	Population size	Number of empires	ε
Symbol	A	B	C
Level 1	50	5	0.1
Level 2	150	10	0.2
Level 3	300	15	0.4

Table 2. The orthogonal array L9.

Experiment	A	B	C
1	−1	−1	−1
2	−1	0	0
3	−1	1	1
4	0	−1	0
5	0	0	1
6	0	1	−1
7	1	−1	1
8	1	0	−1
9	1	1	0

4.3.1 *Ideal distance (ID)*

This metric estimates distance between solutions and best reference point (0, 0). The equation of *ID* for the *i*th solution is defined as following:

$$ID_i = \sqrt{\sum_{k=1}^{nf} Nf_{ik}^2}.$$

where Nf_{ik} is a normalised value of the kth response function of the ith obtained solution. The lower the value of MID in a minimising problem, the better solution quality we have.

4.3.2 *Improvement (IMP)*

We evaluate the proposed algorithm by relative improvement with respect to the mentioned methods (Karimi *et al.* 2010). For a minimising problem, this metric can be calculated as below:

$$\text{improvement} = \frac{\sum_{k=1}^{nf} \frac{f_k(MA) - f_k(PA)}{f_k(MA)}}{nf} \times 100$$

Here $f_k(MA)$ and $f_k(PA)$ are the kth response value obtained through the mentioned algorithm and the proposed algorithm, respectively, and nf is the number of responses.

4.3.3 *Relative closeness to ideal solution for each solution (cl_i)*

This measure is used to sort the solutions in terms of similarity based on cl_i, from the most similar to the least. The least close solution is best.

5. Experimental results

In this section we try to evaluate the efficiency and effectiveness of the proposed algorithm. Since there is not any reliable benchmark in this field we do the assessment utilising three different case studies from three published papers. We apply our proposed algorithm on their RSM-based models and compare the obtained results to check the performance of the algorithm. In the following we present a brief description of each case study and then do the comparisons.

Case study 1: In this example, we like to test the proposed algorithm against a nonlinear multi-response model. Rahman *et al.* (2007) studied the effect of H_2SO_4 concentration, reaction temperature and reaction time for the production of xylose. They used a rotatable central composite design (CCD) in order to fit a second order model, and RSM was utilised to optimise the hydrolysis process in order to obtain high xylose yield. The model is as follow:

$$\begin{aligned} Y_1 = {} & 86.94 + 2.41x_1 + 3.83x_2 + 6.99x_3 - 19.15x_1^2 - 7.09x_2^2 - 12.47x_3^2 - 11.15x_1x_2 \\ & - 22.08x_1x_3 - 1.15x_2x_3 \\ Y_2 = {} & 16.38 - 3.55x_1 - 4.05x_2 + 0.35x_3 - 4.4x_1^2 + 2.53x_2^2 - 2.34x_3^2 - 0.17x_1x_2 \\ & - 5.31x_1x_3 - 0.51x_2x_3 \end{aligned}$$

where Y_1 stands for xylose yield and Y_2 selectivity, x_1 temperature, x_2 reaction time and x_3 acid concentration. See Rahman *et al.* (2007) for more details.

As can clearly be seen from Table 3, the proposed method outperforms the mentioned algorithms in all of the performance metrics. Note that in a maximisation problem, bigger *ID* is better. This result can also be seen from Figure 5(a) and Figure 5(b).

Case study 2: This case study that strives to optimise the bi-response optimisation problem was considered by Kuar *et al.* (2006). They fitted models for minimising HAZ and taper condition during pulsed Nd:YAG laser

Table 3. Comparison of four algorithms through three different metrics on case study 1.

Methods	X_1	X_2	X_3	Y_1	Y_2	ID	$Imp.$	Cl_i
Pasandideh and Niaki	−0.4583	−0.7775	0.5074	76.5009	22.7116	0.6846	1.863%	0.7647
S. Fan *et al.*	−0.2692	−0.9870	0.5217	75.1996	23.9877	0.6974	0.067%	0.3405
Fallah-Jamshidi *et al.*	0	−1	0.3	77.3397	23.0074	0.6928	0.689%	0.5758
Proposed method	−0.236	−1	0.5015	75.3122	23.9840	0.6978		0.3291

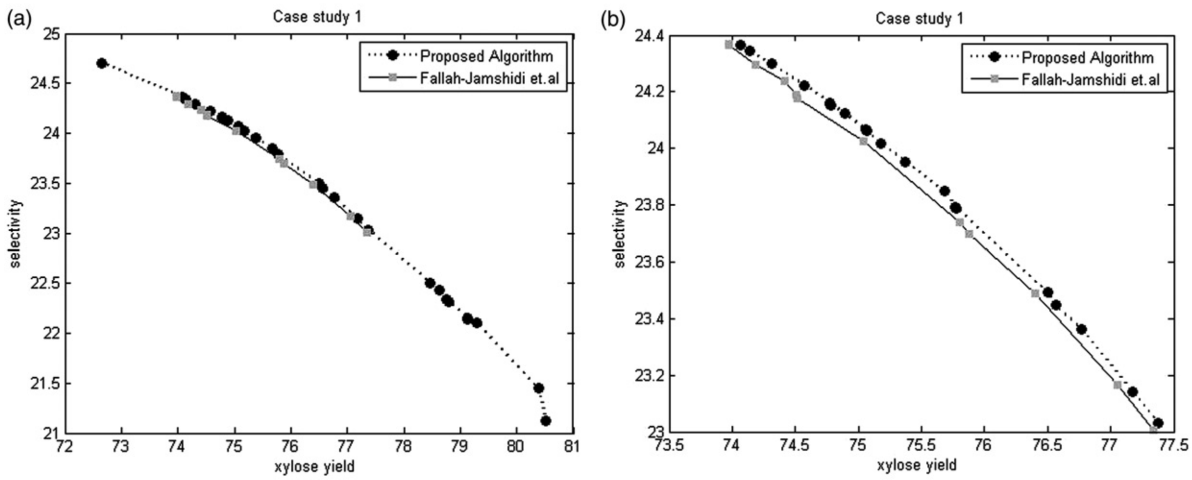

Figure 5. (a) Response function values of case study 1. (b) Magnification of mentioned area.

Table 4. Comparison of four algorithms through three different metrics on case study 2.

Methods	x_1	x_2	x_3	x_4	Y_1	Y_1	ID	$Imp.$	cl_i
Pasandideh and Niaki	−1.9886	−0.3295	−1.9920	−1.9910	0.1949	−0.2459	0.5000	39.061%	0.6707
S. Fan *et al.*	−1.8946	0.9943	−1.9892	−1.9892	0.1860	−0.2365	0.4776	36.438%	0.6785
Fallah-Jamshidi *et al.*	−2	2	−2	−2	0.1471	−0.2331	0.3783	29.043%	0.4549
Proposed method	1.0415	2	2	2	0.0963	−0.1782	0.2828		0.3293

micro-drilling on ZrO2 using a central composite rotatable second-order design. These models are as follows:

$$\begin{aligned} Y_1 = {} & 0.3796 + 0.07888x_1 - 0.0412x_2 - 0.04301x_3 - 0.0057x_4 + 0.02146x_1^2 - 0.00957x_2^2 + 0.00266x_3^2 \\ & - 0.01234x_4^2 - 0.02228x_1x_2 - 0.00679x_1x_3 - 0.03158x_1x_4 + 0.01341x_2x_3 - 0.00983x_2x_4 - 0.00497x_3x_4 \\ Y_2 = {} & 0.07253 + 0.00912x_1 + 0.00887x_2 - 0.00606x_3 + 0.00449x_4 + 0.00153x_1^2 + 0.00225x_2^2 + 0.00233x_3^2 \\ & + 0.00399x_4^2 + 0.00431x_1x_2 - 0.00646x_1x_3 - 0.00519x_1x_4 - 0.0011x_2x_3 - 0.00023x_2x_4 - 0.07253x_3x_4 \end{aligned}$$

where Y_1 represents thickness of HAZ, Y_2 taper of the machined hole, x_1 lamp current, x_2 pulse frequency – that is to say frequency of Q-switch – x_3 air pressure and x_4 pulse width. See more details in Kuar *et al.* (2006).

Table 4 and Figure 6 also show that the proposed algorithm attained the best results.

Case study 3: In the current investigation, which is considered by Onwubolu (2006), there are three variables, x_1, x_2 and x_3. The following independent controllable process parameters were identified to carry out the experiments: drill speed (x_1), drill feed rate (x_2) and drill bit diameter (x_3). Trial runs were carried out by varying one of the process parameters whilst keeping the rest of them at constant values. The aim was to minimise two responses (axial force,

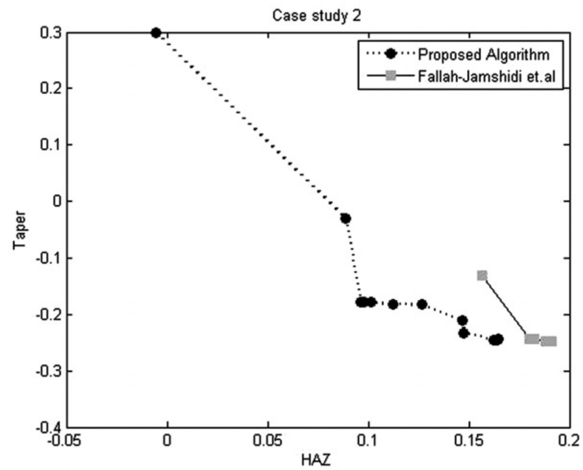

Figure 6. Response function values of case study 2.

Table 5. Comparison of four algorithms through three different metrics on case study 3.

Methods	x_1	x_2	x_3	Y_1	Y_2	*ID*	*Imp.*	cl_i
Pasandideh and Niaki	0.9909	0.5138	−0.9792	36.8443	0.7053	0.6649	12.92%	0.7662
S. Fan *et al.*	0.9632	0.6179	−0.9416	38.7132	0.6839	0.6687	14.74%	0.8167
Fallah-Jamshidi *et al.*	0.9974	0.4975	−0.9896	36.4708	0.7056	0.6621	12.35%	0.7536
Proposed method	1	1	−1	42.0102	0.4241	0.5834		0.2464

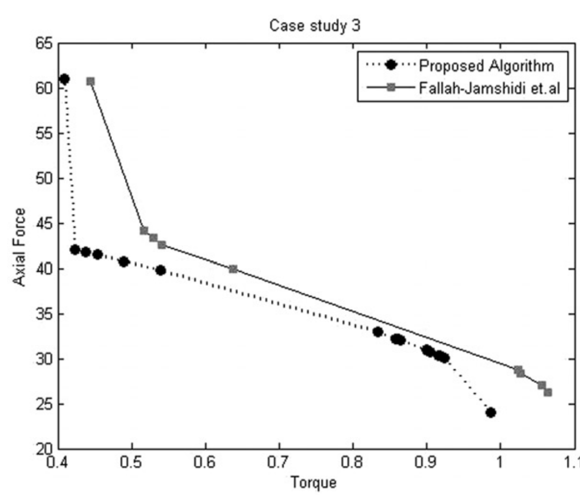

Figure 7. Response function values of case study 3.

P, and torque, T). The mathematical relationships for correlating responses and the considered process variable areas follow. For more details see Onwubolu (2006).

$$Y(P) = 51.6045 - 6.7395x_1 + 9.0524x_2 + 11.9810x_3 + 2.4695x_1^2 + 1.5858x_2^2 - 0.1815x_3^2 \\ - 1.25x_1x_2 + 3.75x_1x_3 - 1.2x_2x_3$$

$$Y(T) = 1.2258 + 0.0124x_1 - 0.0364x_2 - 0.0478x_3 - 0.1785x_1^2 - 0.1785x_2^2 - 0.1919x_3^2 \\ - 0.0908x_1x_2 + 0.0317x_1x_3 + 0.1542x_2x_3$$

6. Conclusion and future work

In order to tackle multi-response optimisation difficulties, we proposed a new hybrid multi-objective ICA to investigate the nonlinear continuous multi-response problems. A simulation approach was used to generate diverse initial solutions. A PSO-like mechanism was introduced to simulate imperialists' assimilation strategy. If there are multiple non-dominated solutions, a well-known classical MCDM method (that is to say TOPSIS) would be used to identify which ones are imperialists and which ones belong to the colonies' society.

To validate the effectiveness of the proposed multi-objective ICA, we have considered three distinguished case studies from published papers and have evaluated the performance and the reliability of the proposed algorithm in comparison with mentioned works in the literature. Different comparison metrics, such as the rate of achievement to two responses simultaneously, mean ideal distance, improvement and relative closeness to ideal solution, were applied to appraise the efficiency of the proposed algorithm. As evidenced from the illustrations, the proposed algorithm considers more expanded ranges of searching space than other existing algorithms. The experimental results reveal that the proposed hybrid multi-objective ICA outperformed other methods. In all case studies, the multi-objective ICA was able to improve the quality of the obtained solutions.

Future research directions involve the consideration of other meta heuristics for this problem and other methods for data generation instead of simulation.

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