

## Thermal Effect on Buckling of General Dome Ends Using Finite Element Method

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**Abstract.** In this study, the elastic buckling behavior of general dome ends under presumed temperature distribution and external pressure was studied. The Finite Element Method (FEM) was used to predict the elastic buckling pressure behavior when the domes were subjected to various operating temperatures. The freedom of the edges of the dome ends was completely restricted to simulate clamped end conditions. The four-centered ellipse method was used to construct the geometry of the dome ends. The influence of geometrical parameters such as thickness, knuckle radius, and the ratio of minor axis to the major axis of dome ends and the effect of temperature on critical buckling pressure of hemispherical, ellipsoidal, and torispherical dome ends were studied. It has been found that the under thermal condition, the thickness and the shape of the domes have the most significant effect on the critical buckling pressure. Two models of torispherical and ellipsoidal dome ends are analyzed by using finite element analysis.

### Introduction

Dome ends conspicuously find many applications in industries. A dome end can become unstable and buckle when the thermal loading, with or without initial mechanical loads, reaches a certain amount. The critical thermal and mechanical parameters are their simultaneous magnitudes at which the structural element tends to buckle. Abdi, et al., [1]

Hemispherical, ellipsoidal, and torispherical domes are three types of dome ends that are used commonly in pressure vessels. The results of previous studies are reported in [2-8]. Thornton studied thermal buckling of plates and shells [9]. Shell-plate interaction in free vibrations of circular cylindrical tanks partially filled with a liquid was studied by Amabili [10]. By using semi-analytical finite element method, Kochupillai et al., [11] suggested a new formulation for elastic shells conveying fluids. Teng worked on buckling of thin shells that have geometric imperfection, due to uniform loading, wind and earthquake loads [12]. In the review articles that deals on finite element analysis of pressure vessels and pipes by Mackerle [13], it can be seen that in the last decade there are a few reported researches on the thermal effects on buckling of general domes by using Finite Element Method.

In this present study, hemispherical, ellipsoidal, and torispherical dome ends are three common types of dome ends that are selected and analyzed (Fig. 1). Two models each of hemispherical, ellipsoidal, and torispherical dome ends are selected and analyzed. All models are studied in three various temperatures and the effects of temperature on buckling of the domes are studied. The dimensions of the dome ends used for the analysis are same as those reported in the work of Yang et al., [14]. All domes are rigidly supported at the edges. The domes are thin and elastic buckling analyses are used in this study. To construct the torispherical dome ends, the four-centered ellipse method was used [15]. Together with the shape of the dome ends, the effects of wall thickness on the critical external buckling pressure under three various temperatures are studied.

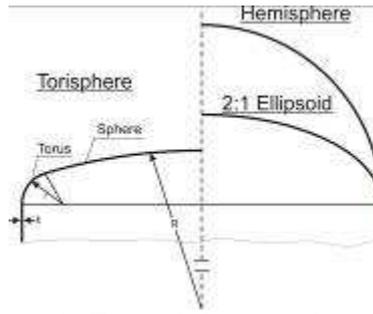


Figure 1: Typical dome end profiles

**Geometric description of torispherical dome ends**

Consider a torispherical dome end with constant thickness  $t$  under external pressure. The edges have no flanges and are assumed to be fully clamped to a fixed support. The geometry of torispherical dome end can be constructed by using the four-centered ellipse method (Fig. 2). If the dome has a geometric ratio  $K$  (ratio of minor axis  $b$  to major axis  $a$ ), then the other parameter  $\theta, \phi, r$ , and  $R$  can be determined from the following equations:

$$\theta = \tan^{-1}(b/a) = \tan^{-1} K, \quad \phi = \pi/2 - \theta \tag{1}$$

$$r = \frac{a}{2} \left[ \frac{1 + \sin \theta - \cos \theta}{\cos^2 \theta} \right], \quad R = \frac{a}{2 \sin \theta} \left[ 1 + \frac{1 - \sin \theta}{\cos \theta} \right] \tag{2}$$

where  $R, r$  are the radius of Sphere and torus, respectively and  $\phi, \theta$  are the Angles of Sphere and torus, respectively. The thickness of dome is  $t$  and the manor and major axis of dome are  $a$  and  $b$ , respectively.

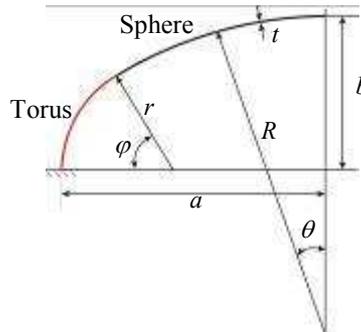


Figure 2: Geometry of torispherical dome end

**Model Description**

Two models each of hemispherical, ellipsoidal and torispherical dome ends were considered. The dimensions of all dome ends used here are same as those reported in the work of Yang et al [14]. Keeping the Poisson’s ratio of 0.3 constant, the elastic modulus was made to vary with temperature, i.e. the modulus of elasticity is a function of temperature ( $201.48 \text{GN} / \text{m}^2$  at  $37.78^\circ \text{C}$ ,  $198.72 \text{GN} / \text{m}^2$  at  $93.33^\circ \text{C}$ ,  $195.27 \text{GN} / \text{m}^2$  at  $148.9^\circ \text{C}$ ). For all types of dome ends, the major axis  $a$  for Model 1 is fixed at  $1.625 \text{m}$  and for Model 2, it is fixed at  $2.330 \text{m}$ . The range of the ratio  $K (= b/a)$  for ellipsoidal and torispherical dome ends are selected as  $0.2 \leq K \leq 0.8$  and the range of thickness  $t$  as  $0.00508 \leq t \leq 0.0508(\text{m})$ . The principal dimensions of ellipsoidal dome end are listed in table 1 and for the torispherical dome ends are listed in Table 2.

Table 1: Principal data of ellipsoidal dome end

Ratio		5:1	3.3:1	2.5:1	2:1	1.67:1	1.4:1	1.25:1
$K = b/a$		0.2	0.3	0.4	0.5	0.6	0.7	0.8
Minor axis, $b(\text{m})$	Model 1	0.325	0.487	0.650	0.812	0.975	1.138	1.300
	Model 2	0.466	0.699	0.932	1.165	1.398	1.631	1.864

Table 2: Principal data of torispherical dome end

$K = \frac{b}{a}$	Toroidal knuckle radius, $r$ (m)		Spherical cap radius, $R$ (m)		Toroidal knuckle angle, $\theta$		Spherical cap angle, $\varphi$	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
0.2	0.182	0.261	7.534	10.801	11.309	11.310	78.690	78.691
0.3	0.292	0.418	4.928	7.065	16.699	16.699	73.300	73.301
0.4	0.417	0.598	3.666	5.256	21.801	21.801	68.198	68.198
0.5	0.561	0.804	2.937	4.212	26.565	26.565	63.434	63.435
0.6	0.725	1.040	2.472	3.543	30.963	30.964	59.036	59.036
0.7	0.912	1.308	2.153	3.087	34.992	34.992	55.008	55.008
0.8	1.124	1.611	1.924	2.759	38.659	38.659	51.340	51.340

### Finite Element Method

The thermal effect on elastic buckling of externally pressurized domes is carried out by using finite element method (FEM). Two models each of hemispherical, ellipsoidal and torispherical dome ends were considered and studied in this paper. The effect of thickness at the onset of buckling at three various temperatures was carried out in this study. Also, the effect of shape of torispherical dome ends at various temperatures on critical buckling pressure was studied because the geometry consists of toroidal and spherical parts. For a value of  $K$ , there exists an infinite combination of  $r$  and  $R$  (two examples in Table 2), all of which have different elastic buckling behavior.

### Results and discussion

For the hemispherical domes, the thickness effect on the buckling pressure at three various temperatures are studied and the results are shown in Fig. 3. In Fig. 4 the thickness effect on the buckling pressure of ellipsoidal domes at three various temperatures are presented. In this figure, the ellipsoidal dome has  $K = b/a = 0.5$  for both models.

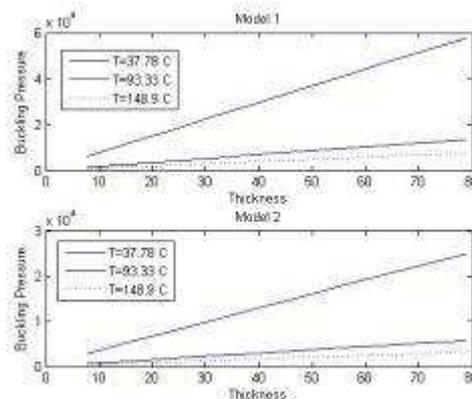


Figure 3: Effect of thickness on buckling pressure, of hemispherical dome ends

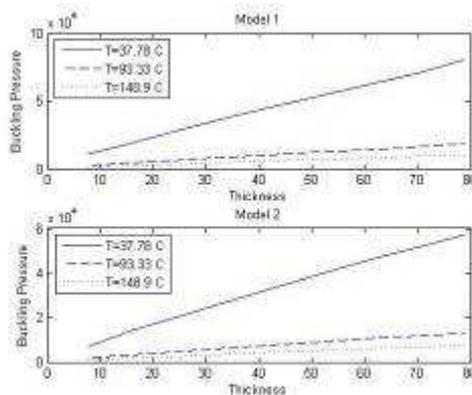


Figure 4: Effect of thickness on buckling pressure, of ellipsoidal dome ends

When thickness is kept constant, the influence of ratio  $K = b/a$  on the buckling pressure of ellipsoidal dome ends at various temperatures is studied and the results are displayed in Fig. 5. In this study, the thickness for Model 1 is 0.032 m and 0.044 m for Model 2.

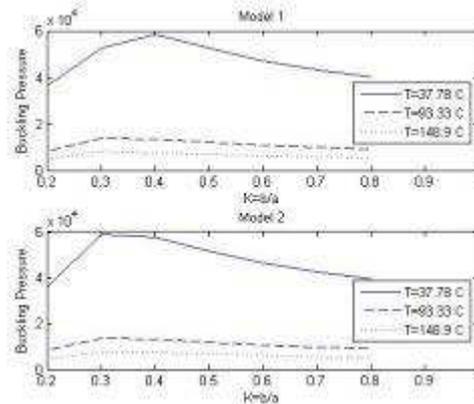


Figure 5: Effect of  $K = b/a$  on buckling pressure, of ellipsoidal dome ends (constant  $t$ )

For torispherical dome end with constant thickness, the influence of ratio  $K = b/a$  on the buckling pressure at various temperatures is studied and the results are displayed in Fig. 6. In this figure, the thickness of torispherical for Model 1 is 0.032 m and 0.045 m for Model 2.

In Fig. 7 the effect of thickness on the buckling pressure of torispherical dome ends at three different temperatures is presented. In this figure, the torispherical dome has  $K = b/a = 0.5$  for both models. Fig. 8.a shows the buckling of a torispherical dome end and Fig. 8.b shows the buckling of an ellipsoidal dome end and with  $K = b/a = 0.5$  and  $t = 0.032 m$  at temperature of  $93.33^{\circ}C$  that was analyzed by finite element analysis software (ANSYS).

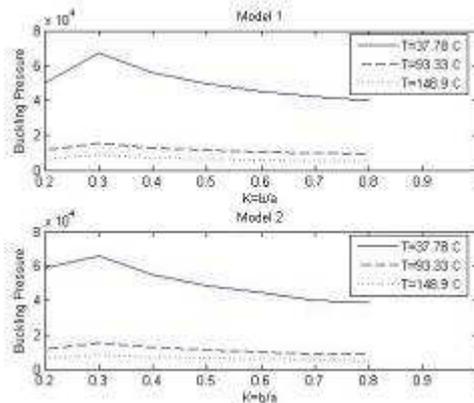


Figure 6: The effect of  $K = b/a$  on buckling pressure, of torispherical dome ends (Constant  $t$ )

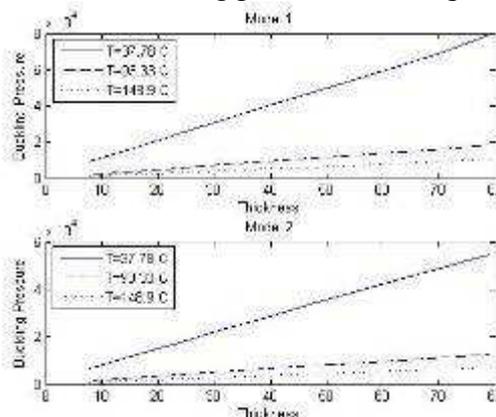
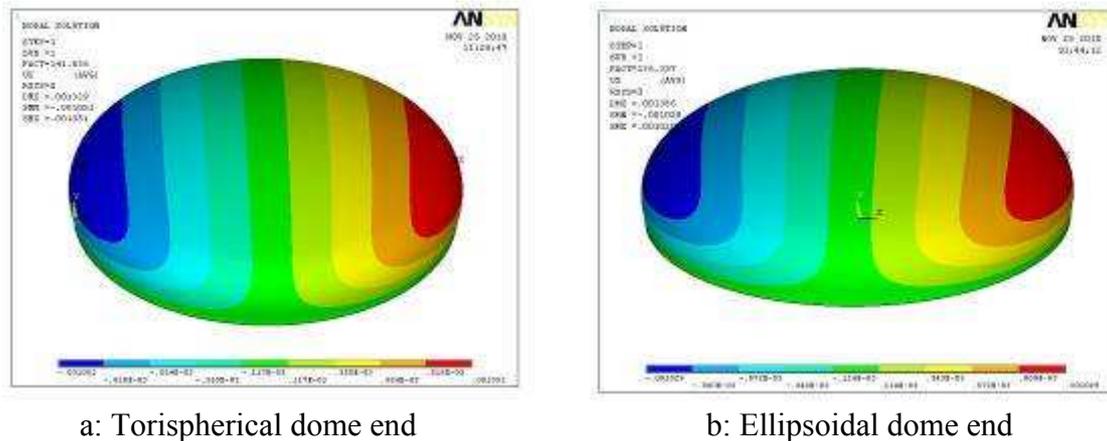


Figure 7: Effect of thickness on buckling pressure, of torispherical dome ends



a: Torispherical dome end

b: Ellipsoidal dome end

Figure 8: Buckling of torispherical and ellipsoidal dome end under thermal and mechanical loading

From Figs. 3-4 and Fig. 7 for hemispherical, ellipsoidal, and torispherical dome ends, it can be seen that the critical buckling pressure increases linearly with thickness. Also in these figures it can be seen that the dome ends are buckled easily at high temperatures. For example the ratio of the critical buckling pressure at  $T = 93.33^\circ\text{C}$  to the critical buckling pressure at  $T = 37.78^\circ\text{C}$  for hemispherical dome end with thickness  $t = 0.032\text{m}$  is 0.23 for Model 1 and Model 2 and this ratio is 0.23 for both models of ellipsoidal and torispherical dome ends.

From Fig. 5, for ellipsoidal dome end, by increasing the ratio  $K (= b/a)$  from 0.2 to the 0.4 for Model 1 at  $T = 37.78^\circ\text{C}$ , the critical buckling pressure increases but then decreases beyond  $K = 0.4$ . The buckling behavior of Model 2 for ellipsoidal dome is similar to Model 1 and the critical buckling pressure increases up until  $K = 0.3$  and after which the critical buckling pressure decreases with increasing  $K (= b/a)$ . At temperatures  $93.33, 148.89^\circ\text{C}$  the buckling behavior of ellipsoidal domes are same as the buckling behavior at  $T = 37.78^\circ\text{C}$ , but the variation of critical buckling pressure is smaller than those at  $T = 37.78^\circ\text{C}$ . The ratio of the critical buckling pressure at  $T = 93.33^\circ\text{C}$  to the critical buckling pressure at  $T = 37.78^\circ\text{C}$  for ellipsoidal dome end with similar thickness and  $K = 0.5$  is 0.23 for Model 1 and Model 2.

For torispherical domes, the effects of thermal loading on the critical buckling pressure are very similar to buckling behavior of ellipsoidal domes under similar loading.

Figs. 5 and 6 show the effect of shape of ellipsoidal and torispherical dome ends on buckling pressure under various thermal and mechanical loading and Fig. 8 show the buckling of ellipsoidal and torispherical domes ( $K = 0.5, t = 0.032\text{m}$ ) that was analyzed by Finite Element Analysis software (ANSYS). It can be seen that in the ellipsoidal and torispherical domes, by increasing the ratio  $K$  from 0.2 to 0.3, the critical buckling pressure increases and then decreases beyond  $K = 0.3$ . From Figs. 5-6, initially increasing the head height (from a flat end cap) will increase the buckling pressure but if the head height is too large, it becomes less resistance again to buckling.

## Conclusion

To define the shape of domed ends, the four-centered ellipse method is an easy method to describe hemispherical, ellipsoidal and torispherical domes. In the presence of a mechanical load, the effects of temperature on the buckling behavior of domes are very important because the dome ends tend to buckle earlier, hence in designing domes, high working temperature condition must be considered. Ellipsoidal and torispherical domes with moderate values of  $K$  have good buckling behavior at high temperatures.

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