A Modified Imperialist Competitive Algorithm for Optimization in Electromagnetics

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Recently, a new kind of socio-politically motivated global search metaheuristic called imperialist competitive algorithm (ICA) was proposed. ICA is based on a form of imperialistic competition in which the populations are represented by countries divided among imperialists and colonies. In this paper, a modified ICA (MICA) approach based on concepts of attraction and repulsion between the colony and its imperialist is introduced during the search for better solutions. A brushless direct current wheel motor benchmark problem is used to investigate the performance of the classical ICA and the proposed MICA and results are shown to be competitive with those of other well-established optimization methods.

***Index Terms*—Electromagnetics, evolutionary computation, optimization.**

I. Introduction

METAHEURISTICS based on evolutionary algorithms and swarm intelligence are known to be powerful methods for the solution of difficult optimization problems related to the design of electromagnetic devices and have been studied extensively in recent years [1]–[6].

A recently introduced evolutionary metaheuristic which has not received much attention in the electromagnetic optimization community is the imperialist competitive algorithm (ICA) [7], which is a global optimization technique based on the behaviour of imperialists in their attempt to conquer colonies. Like other population based algorithms, ICA starts with a randomly generated population of so-called countries. Such countries are divided in two groups according to their objective function values: imperialists, which are the best candidate solutions, and colonies. The main mechanism which leads to the search for better solutions in ICA is the movement of colonies towards the imperialists so that the population tends to converge to certain areas of the search space where the best solution found so far are located.

The objective of this paper is to review the basic algorithmic features of the relatively uncommon ICA optimizer and to present a modified and improved ICA (MICA) variant based on the attraction and repulsion between the colony and its imperialist. Both algorithms are then tested on a brushless direct current (dc) wheel motor benchmark problem described in [2]. Besides, the performance of MICA is compared with that of other metaheuristics presented in the recent literature.

The rest of this paper is organized as follows. Section II describes the ICA and MICA. In Section III, we include a description of the brushless dc wheel motor benchmark problem. Section IV presents the optimization results, and the paper concludes with a discussion in Section V.

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II. Fundamentals of ICA and MICA Algorithms

ICA uses the concepts of imperialism and imperialistic competition process as a source of inspiration. ICA starts with an initial population consisting of *countries* (individuals in other evolutionary algorithms) which are divided in two groups. The ones with the best objective function values are selected to be the *imperialists*, whereas the remaining ones are their *colonies*. The colonies are then shared among the imperialists according to each imperialist's power (objective function value). The more powerful an imperialist is, the more colonies he will possess. In the language of ICA an imperialist with his colonies forms an *empire*.

One of the characteristics of the interaction between imperialist powers and their colonies is that in the course of time colonies start to change their culture in such a way that it becomes more similar to the one of their dominating imperialist. This process is implemented in ICA by moving the colonies towards their imperialist and it is called *assimilation*. During this event there is the possibility for a colony to become more powerful than its imperialist, and in this case the colony will take the place of the imperialist and the imperialist will become one of its colonies.

It can be furthermore observed that during imperialistic competition the most powerful empires tend to increase their power, while weaker ones tend to collapse. These two mechanisms lead the algorithm to gradually converge into a single empire, in which the imperialist and all the colonies tend to have the same culture.

The above description translates into following algorithm.

A. ICA

Step 1: Initialization of the Empires: In ICA, the countries (which are equivalent to the individuals in an evolution strategy) are the feasible solutions in the search space. Each is represented by an array of dimension N_{var} defined by

$$\text{country} = [p_1, p_2, \ldots, p_{N_{\mathrm{var}}}] \tag{1}$$

In the beginning an initial population is randomly created with a uniform distribution and it is divided into imperialists and colonies. Since ICA is a minimization algorithm, the imperialists are usually the countries with the lowest objective function values but since in our case the benchmark problem is a maximization one, the imperialist will be the countries with the highest objective function values.

The colonies will be distributed among the imperialists according to the power of each imperialist. For this purpose, a normalized cost is defined by

$$C_n = c_n - \max_i(c_i) \tag{2}$$

where c_n is the cost of nth imperialist and C_n is its normalized cost. Finally, the normalized power for each imperialist is defined by

$$p_n = \frac{|C_n|}{\left|\sum_{i=1}^{N_{\mathrm{imp}}} C_i\right|}. \tag{3}$$

The normalized power of an imperialist represents the number of initial colonies this imperialist possesses and is given by

$$NC_n = \mathrm{round}(p_n \cdot N_{\mathrm{col}}) \tag{4}$$

where NC_n is the initial number of colonies of the nth empire and N_{col} is the number of colonies.

Step 2: Assimilation—Movement of Colonies Towards the Imperialist: The previously described Assimilation process depends on the distance between the colonies and their respective imperialists, a real constant called assimilation coefficient and a random number in range [0,1]. The following equation represents the new position of a colony:

$$\mathrm{Pos}_{i+1} = \mathrm{Pos}_i + \gamma \cdot \delta \cdot d \tag{5}$$

where Pos_i is the vector of the colony's position on the ith iteration, γ is the assimilation coefficient, δ is a random number normally distributed in range [0,1] and d is a N_{var}-dimensional vector containing the variables distance between the colony and its imperialist. In imperialistic terms, d represents how far the culture of a colony is different from the the culture of the imperialist.

After this movement, it is possible for a colony to reach a position with a lower cost than the imperialist. In this case, this colony will be the new imperialist, whereas the old imperialist will become a colony of the same empire.

Step 3: Evaluation of the Total Cost of Empires: The cost of an Empire is influenced mainly by the imperialist cost, although it is also affected by the costs of the individual colonies. This event is stated by

$$\mathrm{TC}_n = f(\mathrm{Imperialist}_n) + \varepsilon \cdot \mathrm{mean}\{f(\mathrm{Colonies\ of\ Empire}_n)\} \tag{6}$$

where TC_n is the total cost of the nth Empire and ε is a positive constant which represents the significance of the colonies cost.

Step 4: Realization of an Imperialist Competition: The imperialistic competition consists in the dispute between empires in order to conquer the colonies of other empires. This event makes the most powerful empires increase their powers, while the weakest empires tend to decrease their power over time. The imperialistic competition can be modelled by choosing the weakest colony from the weakest empire to be disputed among the other empires. The more powerful an empire is, more chances it will have to conquer the selected colony. For this purpose it is assumed that

$$\mathrm{NTC}_n = \mathrm{TC}_n - \max_i(\mathrm{TC}_i) \tag{7}$$

where TC_n is the total cost of the nth empire and NTC_n is its normalized cost. The possession probability p_{p_n} for the nth empire is given by

$$p_{p_n} = \frac{|\mathrm{NTC}_n|}{\left|\sum_{i=1}^{N_{\mathrm{imp}}} \mathrm{NTC}_i\right|}. \tag{8}$$

Then a vector $\mathbf{P}$ is created with the possession probability of each empire and the vector $\mathbf{R}$, with the same size of $\mathbf{P}$, with random numbers uniformly distributed between [0,1] defined as $\mathbf{P} = [p_{p_1}, p_{p_2}, \ldots, p_{p_{N_{\mathrm{imp}}}}]$ and $\mathbf{R} = [r_1, r_2, \ldots, r_n]$. The vector $\mathbf{D}$ is defined by $\mathbf{D} = \mathbf{P} - \mathbf{R} = [D_1, D_2, \ldots, D_{N_{\mathrm{imp}}}]$. Finally, the empire chosen to possess the colony is the one who has the biggest value of $\mathbf{D}$. This step therefore introduces a randomized algorithmic component.

Step 5: Verification of the Stopping Criterion: Loop to Step 2 until a stopping criterion is met. In this paper, a maximum number of decades is adopted as the stopping criterion.

It should be noted that in typical electromagnetic problems the most expensive part of an optimization run is concentrated in the evaluation of the objective function. In ICA, like in most population-based optimizers, the evaluation is completely parallel and therefore can benefit from modern multicore and multithreaded CPUs or clusters of PCs. A flowchart of the above described basic ICA is shown in Fig. 1.

The standard ICA algorithm, like all stochastic optimization methods, may suffer from premature convergence. This problem arises when the algorithm gets trapped in a local optimum region due a poor exploration of the search space. A method to contrast premature convergence which proved to be successful in the context of particle swarm optimization (PSO) is to introduce alternating attraction and repulsion phases [8]. In PSO algorithms, the repulsion phase is activated when the population (swarm) has a low diversity and the individuals (particles) are repelled in order to better explore the search space. When the population reaches a high diversity it switches back to the attraction phase and the individuals return to converge.

Based on the same general ideas, the above described ICA algorithm can be modified (MICA) by introducing attraction and repulsion between each colony and its imperialist.

B. Proposed MICA

Diversity guided attraction and repulsion can be introduced in ICA by varying the assimilation coefficient according to the distance between the colony and its imperialist.

It has been observed that for $\gamma = 2$ ICA will always converge towards the imperialist [7]. However, if γ is greater than

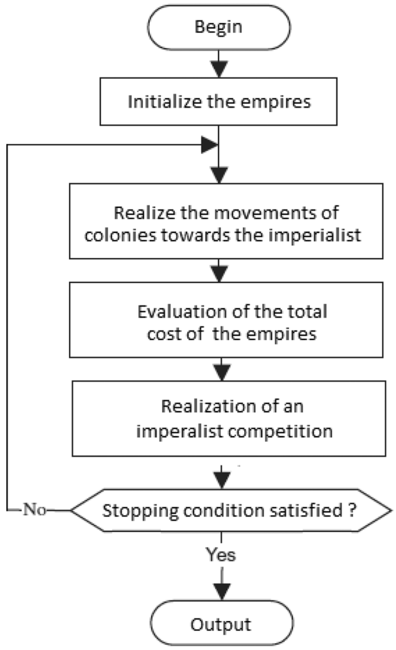

Fig. 1. Flowchart of the basic ICA.

two, a possibility for the colony to move away from the imperialist is created, which results in a better exploration of the search space. Therefore, by increasing γ for a value greater than two, the chances for the colonies to diverge from the imperialist will increase. Furthermore this feature can be introduced in a self-adaptive way by steering the value of γ with the diversity according to the following equation:

$$\gamma = \begin{cases} 2, & \text{if } d > d_{\text{div}} \\ \gamma_{\text{div}}, & \text{if } d \leq d_{\text{div}} \end{cases} \tag{9}$$

where d is a vector containing the distances between the colonies and the imperialist, d_{div} is a distance threshold value and γ_{div} is a number greater than 2.

III. Brushless DC Wheel Motor Benchmark

A brushless dc wheel motor benchmark was presented in [9] and several optimization results obtained with various algorithms are available in literature. Furthermore, the source code for computing the objective function is publicly available [10], thus making the comparison independent of differences in the calculation of the objective function. These features make this benchmark ideal for comparing the performances of different techniques.

The problem is characterized by five continuous design variables (Table I summarizes the degrees of freedom and their range), and the efficiency η of the motor is to be maximized (which is equivalent to minimizing the motor losses). Furthermore, the problem is subject to six inequality constraints which are related to technological and operational considerations regarding the specific wheel motor.

Constraints are handled by a penalty method in the ICA and MICA approaches.

TABLE I
Optimization Variables and Ranges

Variable	Meaning	Minimum value	Maximum value
D_s [m]	Bore (stator) diameter	0.15	0.33
B_e [T]	Air gap induction	0.50	0.76
δ[A/m^2]	Conductor current density	2.0x10^6	5.0x10^6
B_d [T]	Teeth magnetic induction	0.9	1.8
B_{cs} [T]	Stator back iron induction	0.6	1.6

TABLE II
Simulation Results of η in 30 Runs

	η in %			
Method	Maximum (Best)	Mean	Minimum (Worst)	Standard Deviation
ICA	95.30	88.71	46.84	0.83
MICA	**95.32**	**95.18**	**94.68**	0.99

TABLE III
Results of Optimization Using Different Optimization Methods

Optimization method	η (%)	Evaluations of η
SQP	95.32	90
GA	95.31	3380
GA & SQP	95.31	1644
ACO	95.32	1200
PSO	95.32	1600
ICA	95.31	800
MICA	95.32	800

IV. Optimization Results

In order to investigate the applicability and effectiveness of the basic ICA and the proposed MICA to the brushless dc wheel motor benchmark problem, 30 independent runs are performed.

In all runs of ICA and MICA the number of countries was set to 20, the number of imperialists was set to five and the stopping criterion was 40 decades (800 objective function evaluations in each run).

Table II reports the results obtained by ICA and MICA. In this comparison MICA results as a clear winner by providing much better average and worst case results. It can be therefore concluded that the proposed diversity-guided attraction and repulsion mechanism is successful in improving the performance of the algorithm.

Comparisons with other stochastic techniques are possible for this benchmark. Table III shows a comparison between ICA, MICA and a number of well-known deterministic [sequential quadratic programming (SQP)] and stochastic optimizers (genetic algorithm (GA) [12], ant colony optimization (ACO) [13], and PSO [13]).

It can be noted that MICA outperformed all other stochastic algorithms, reaching the global optimum with a much lower number of objective function evaluations.

In terms of the variables corresponding to the optimal motor configuration, the results are detailed in Table IV. It can be noted that the MICA approach converged to the same solution found

TABLE IV
COMPARISON OF RESULTS USING DIFFERENT OPTIMIZATION METHODS

Optimization method	D_s	B_e	δ	B_d	B_{cs}	η	M_{tot}	I_{max}	D_{int}	D_{ext}	T_a
	mm	T	A/mm^2	T	T	%	kg	A	Mm	mm	^{o}C
SQP [11],[12]	201.2	0.6481	2.0437	1.8	0.8959	95.32	15	125	76	238.9	95.35
GA [12]	201.5	0.6480	2.0602	1.799	0.8817	95.31	15	125	76.9	239.2	95.21
GA & SQP [12]	201.2	0.6481	2.0615	1.8	0.8700	95.31	15	125	76	238.9	95.31
ACO [13]	201.2	0.6481	2.0437	1.8	0.8959	95.32	15	125	76	238.9	95.35
PSO [13]	201.1	0.6476	2.0417	1.8	0.9298	95.32	15	125	79.2	239.8	94.98
Proposed MICA	201.2	0.6481	2.0437	1.8	0.8959	95.32	15	125	76	238.9	95.35

by SQP and ACO, which is most probably the global optimum of the optimization problem.

V. CONCLUSIONS

In this paper, the performance of the standard ICA and a modified variant of the algorithm (MICA) have been evaluated on a typical electromagnetic optimization problem. The modified algorithm is shown to be much more robust than the standard algorithm, thus providing much better average and worst case solutions. The best solutions found in a batch of runs are competitive with that of other well-established optimization techniques in spite of the relative novelty of the algorithm. Further benchmarking on other common electromagnetic problems is currently under way. The proposed MICA is also being extended to multi-objective optimization problems. Parallel or distributed versions are also being considered for future implementations.

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